Safety-Critical Machine Learning: Development & Testing

Aman Sinha
August 17, 2020

Motivation

- We are starting to apply machine learning to high-stakes decision-making



- The standard paradigms of ML aren't enough in safety-critical applications
  - Development: minimize average loss over a nominal training dataset
  - Testing: check average performance over a test dataset

- New paradigms for handling uncertainty
  - Development: build **robustness** against uncertainties
  - Testing: quantify **risk** (likelihood and severity) of failures

Key ideas of this talk

Robustness

Build models with guaranteed performance over uncertainty sets

Risk

Find failure modes and quantify the probability of failure

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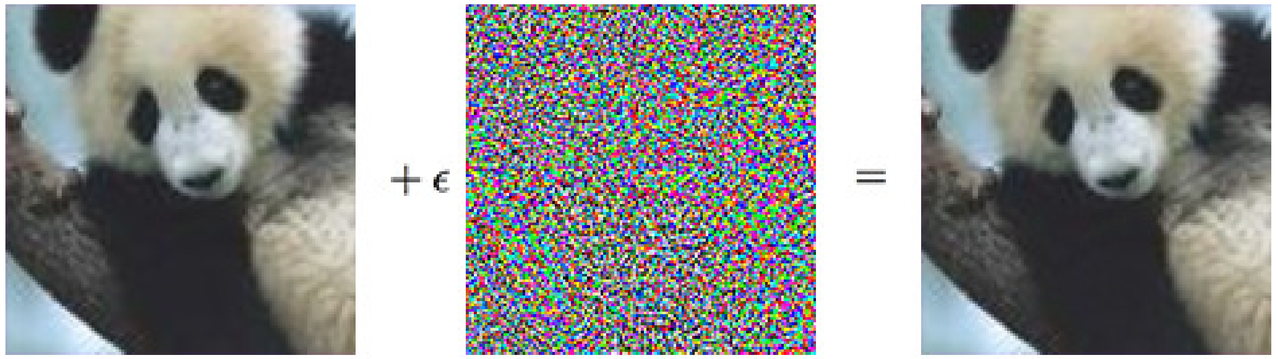
Certifiable robustness against adversarial attacks

Sinha*, Namkoong*, Duchi. ICLR 2018.
Sinha*, Namkoong*, Volpi, Duchi. Under review.



Certifiable robustness against adversarial attacks



[Goodfellow et al. 2015]

[Athalye et al. 2017]

We want to increase the robustness of ML systems to adversarial attacks (small $\mathcal{P}$)

Current approaches

- Adversarial training heuristics: fast but no theoretical guarantees of robustness
  - Goodfellow et al. 2015, Kurakin et al. 2016, Papernot et al. 2016, He et al. 2017, Carlini & Wagner 2017, Tramer et al. 2017, Madry et al. 2018, etc.

- Formal verification: rigorous guarantees but slow
  - Huang et al. 2017, Katz et al. 2017, Kolter & Wong 2017, Tjeng & Tedrake 2017, Raghunathan et al. 2018

Our goal: balance efficiency with robustness guarantees

Our work: principled adversarial training

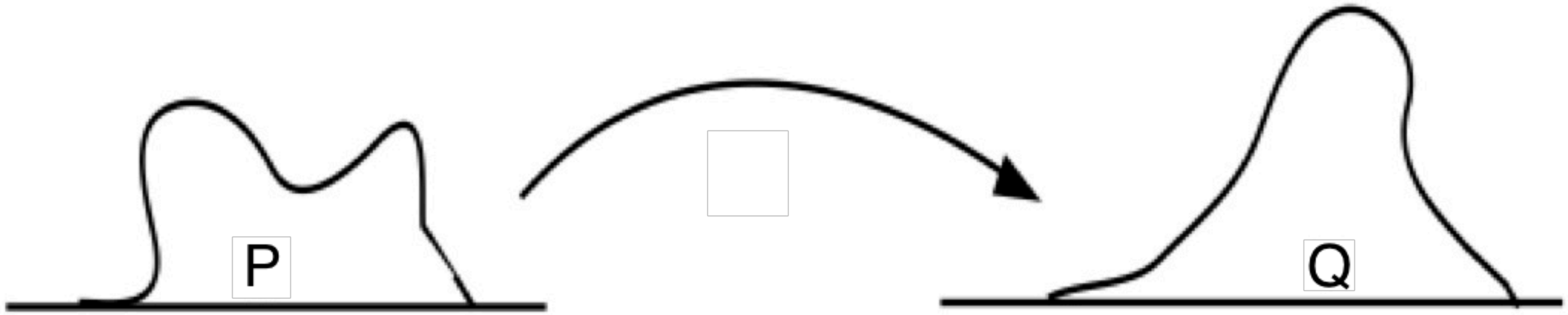
- Setup: model/network weights $\theta \in \Theta$, feature vector X, label Y, and loss $\ell(\theta; X, Y)$

Overall idea: replace $\ell(\theta; X, Y)$ with robust surrogate $\phi_\gamma(\theta; X, Y)$

- For moderate levels of desired robustness and smooth losses ℓ:
  - Provably fast convergence, 5-10x as fast as ERM
  - Statistical guarantees for performance on (perturbations to) the test set

Distributionally robust optimization (DRO)

- Goal: robustness to perturbations in a Wasserstein ball
- Generally intractable for arbitrary ρ



[Esfahani & Kuhn 2015; Shafieezadeh-Abadeh et al. 2015; Blanchet et al. 2016, Lee & Raginsky 2017]

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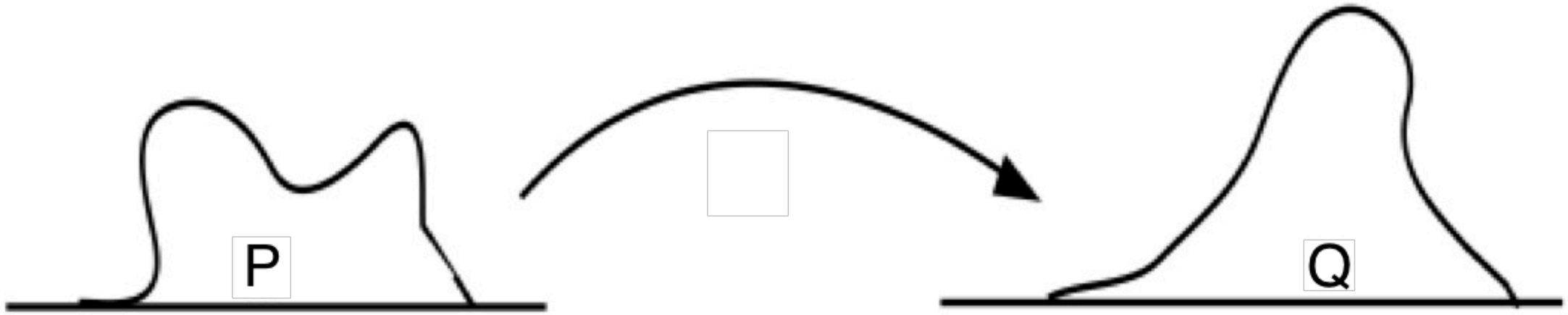

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Distributionally robust optimization (DRO)

$$\underset{\theta \in \Theta}{\text{minimize}} \quad \max_{Q} \{\mathbb{E}_Q[\ell(\theta; X, Y)] : D_c(Q, P_0) \leq \rho\}$$

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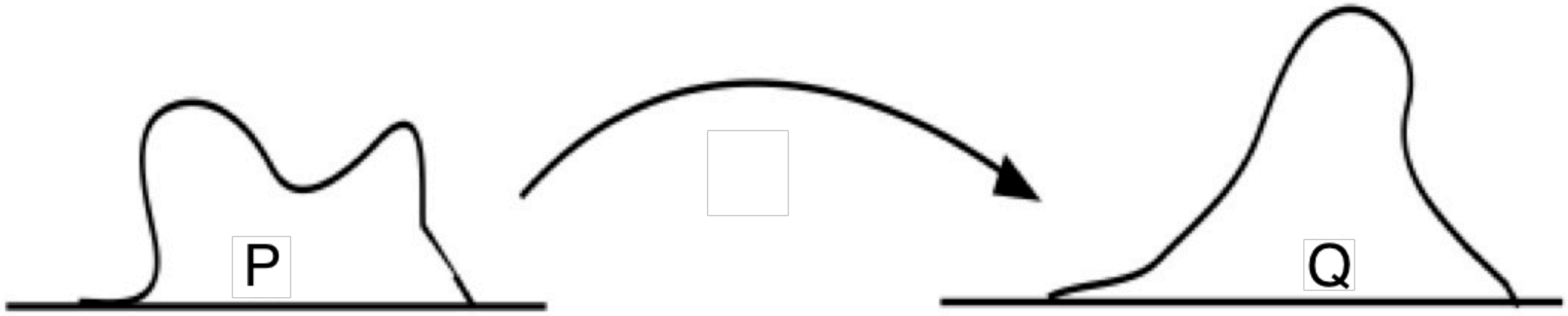


[Esfahani & Kuhn 2015; Shafieezadeh-Abadeh et al. 2015; Blanchet et al. 2016, Lee & Raginsky 2017]

Distributionally robust optimization (DRO)

- Lagrangian relaxation and its dual formulation
  - More robustness ↔ larger ρ ↔ smaller γ

$$\underset{\theta\in\Theta}{\text{minimize}} \ \max_{Q} \left\{ \mathbb{E}_Q[\ell(\theta; X, Y)] - \underbrace{\gamma D_c(Q, P_0)}_{\text{penalty}} \right\} =$$

$$\underset{\theta\in\Theta}{\text{minimize}} \ \mathbb{E}_{P_0}[\phi_\gamma(\theta; X, Y)]$$

$$\text{where} \ \ \phi_\gamma(\theta; x, y) := \max_{x'\in\mathcal{X}} \left\{ \ell(\theta; x', y) - \underbrace{\gamma \|x' - x\|^2}_{\text{penalty}} \right\}$$

- Compare to ERM: $\text{minimize}_{\theta\in\Theta} \ \mathbb{E}_{P_0}[\ell(\theta; X, Y)]$

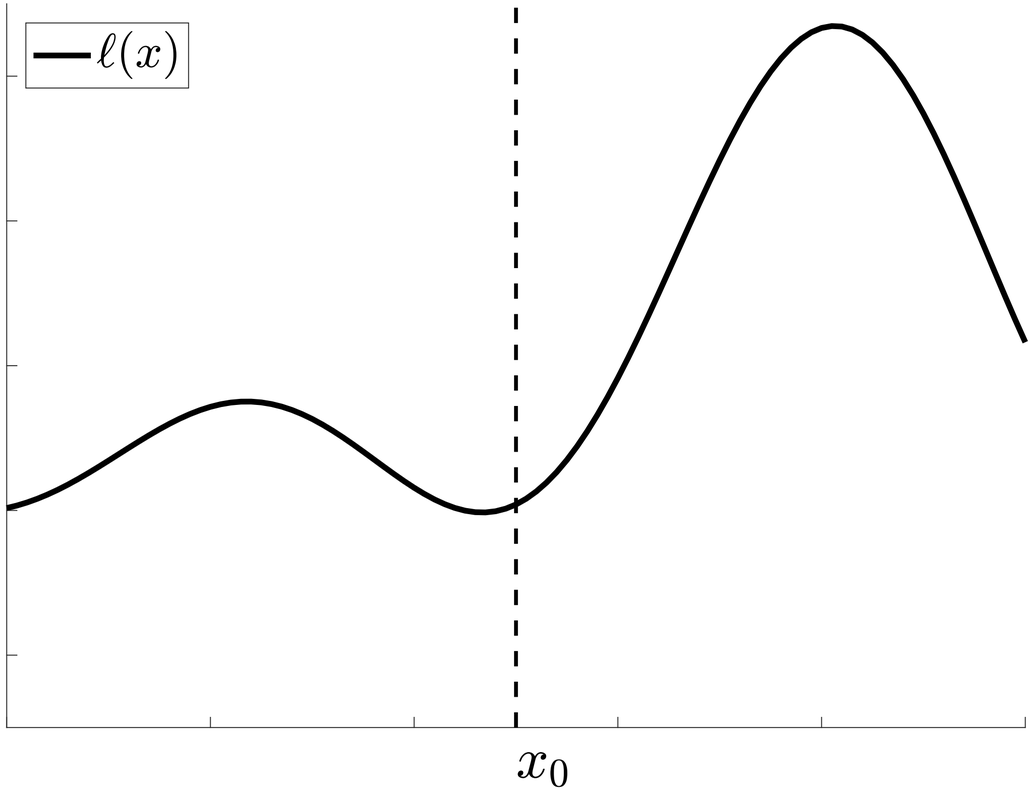
[Blanchet et al. 2016]

Solving the optimization problem

$$\phi_\gamma(\theta; x_0, y_0) := \max_{x \in \mathcal{X}} \left\{ \ell(\theta; x, y_0) - \gamma \|x - x_0\|^2 \right\}$$

Key insight: $(x, y) \mapsto \ell(\theta; x, y) - \gamma \|x - x_0\|^2$ is strongly concave for **smooth** ℓ and large enough γ

- Curvature in $\|\cdot\|^2$ dwarfs out non-concavity of $\ell(\theta; \cdot)$



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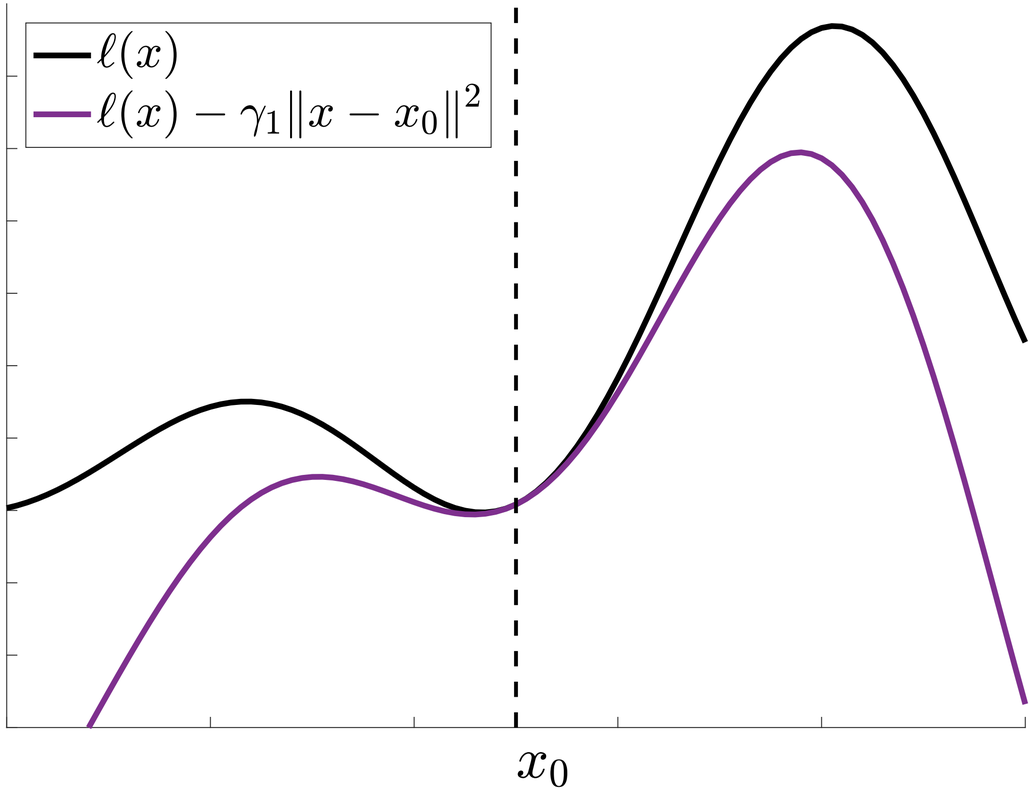

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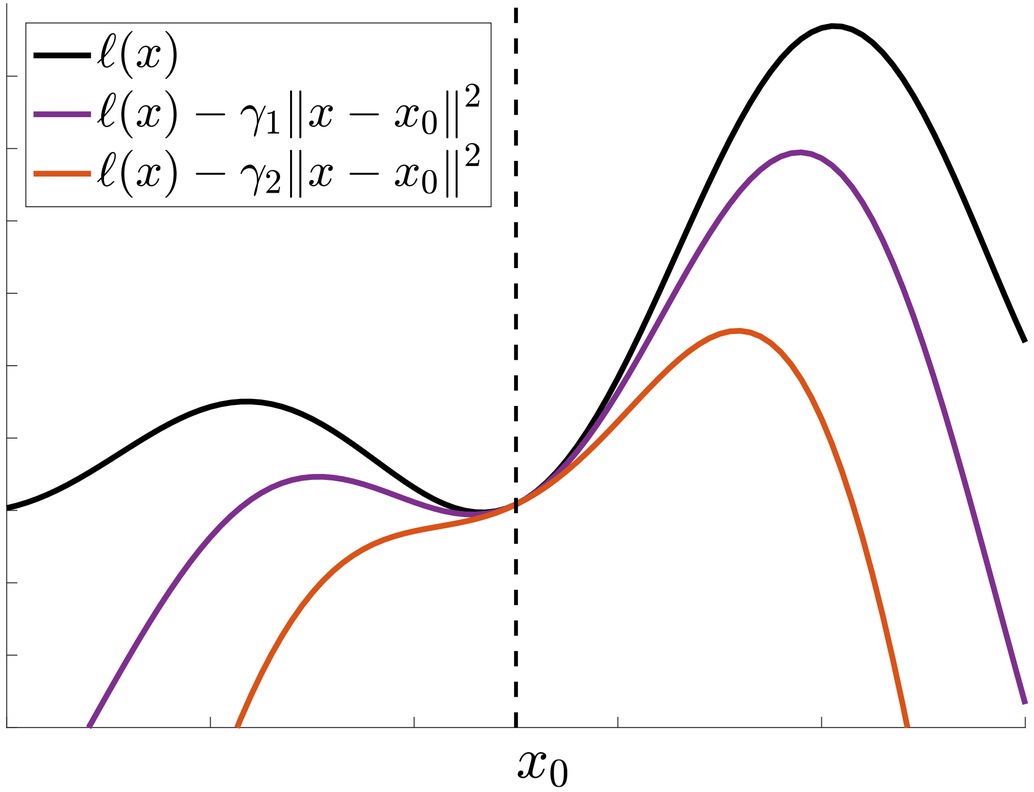

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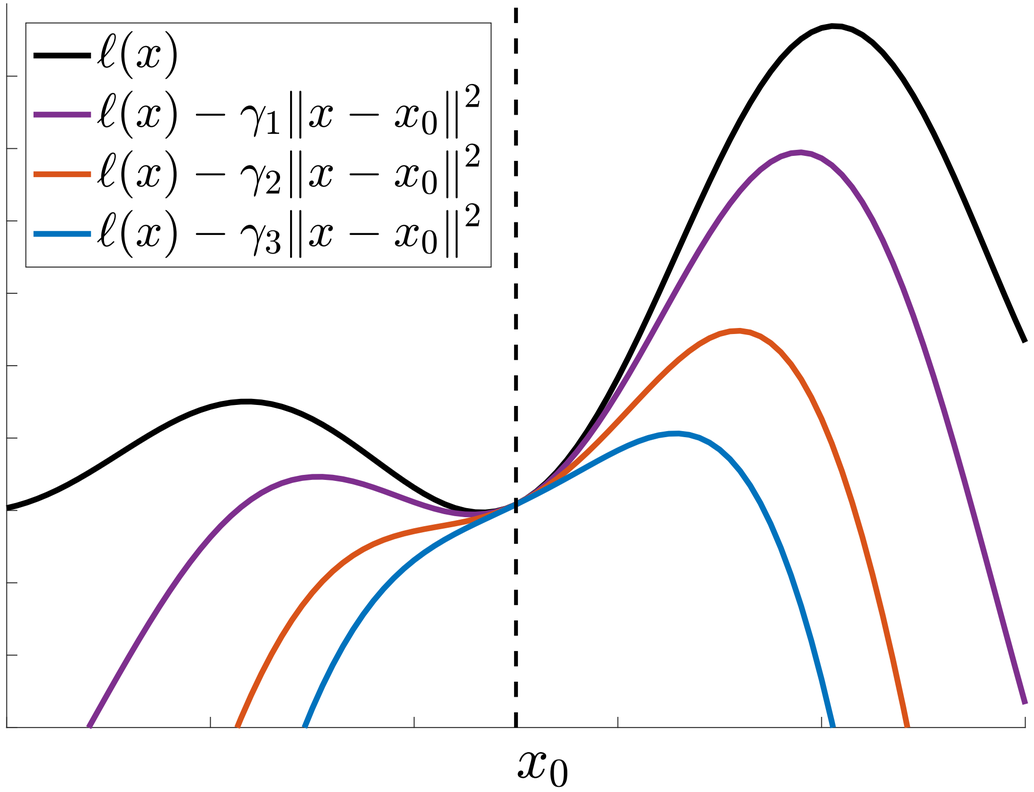

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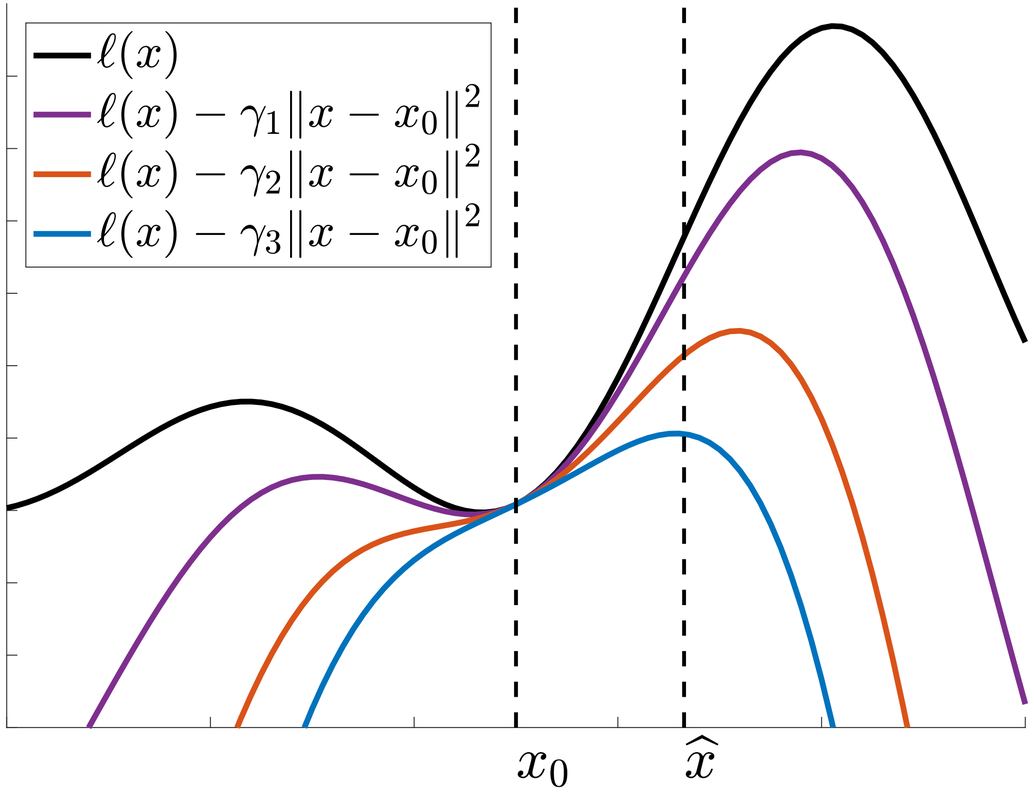

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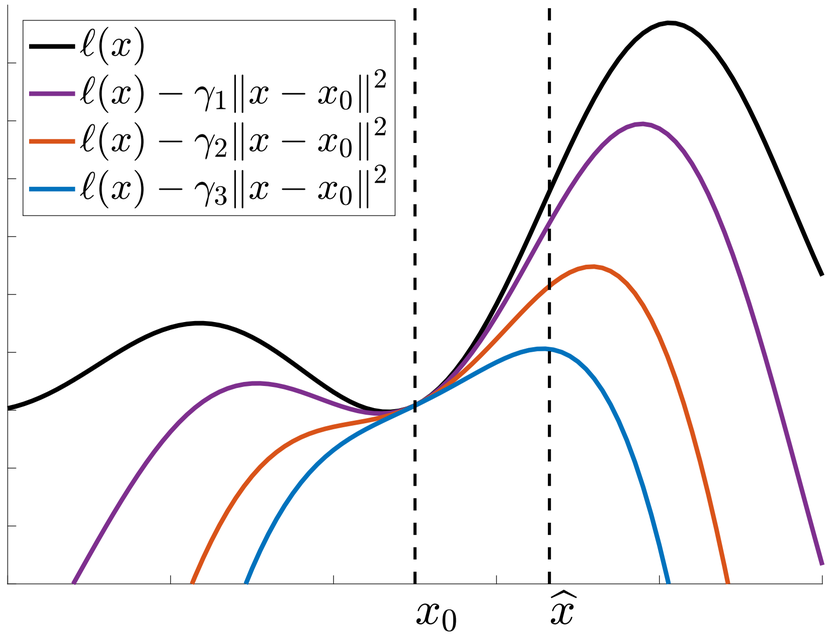


Deep nets with smooth activations (ELUs, sigmoid, etc.) are smooth

Optimization guarantees

Algorithm: SGD for $\min_\theta \mathbb{E}_{P_0}[\phi_\gamma(\theta; X, Y)]$

- Sample $(x^t, y^t) \sim P_0$
- Compute adversarial example:
  (approximate) maximizer $\widehat{x}^t$ of $\ell(\theta^t; x, y^t) - \gamma\|x - x^t\|^2$
- $\theta^{t+1} \leftarrow \theta^t - \alpha\nabla_\theta \ell(\theta^t; \widehat{x}^t, y^t)$

- For large enough γ we can compute $\widehat{x}^t$ in 10-20 gradient ascent steps

- Theorem: converges at standard nonconvex-SGD rate

Certificate of robustness

- Algorithm **generalizes**: we learn to prevent attacks on the **test** set

- $\theta_{\mathrm{WRM}} =$ output of Algorithm, $\mathfrak{Comp}_n =$ size of Θ, $C =$ problem-dependent constant, $\widehat{P}_n =$ empirical training distribution

Theorem (Robustness Certificate)

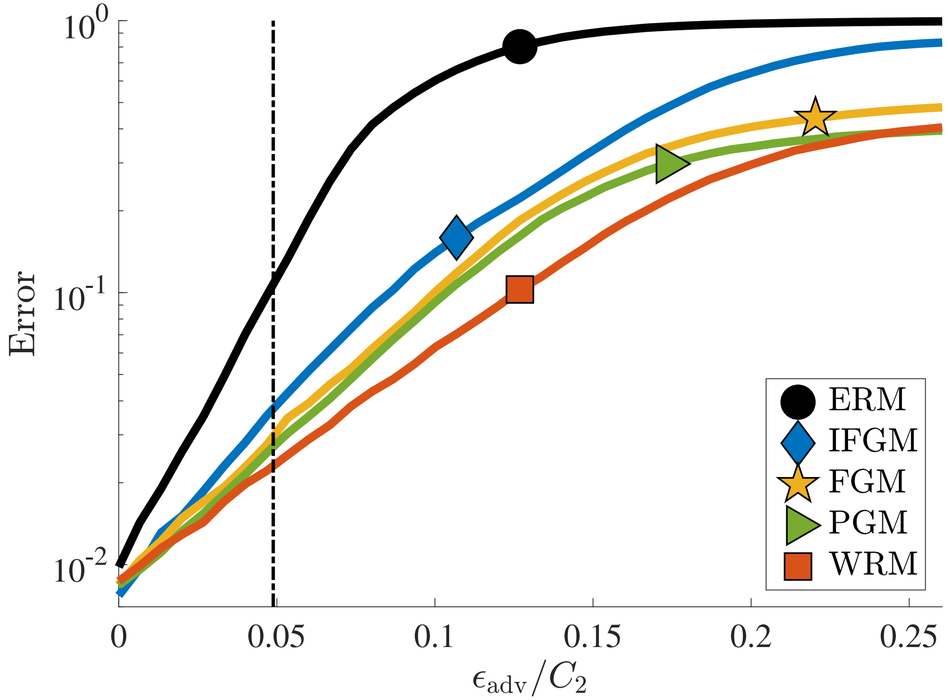
With high probability, for any $\rho \geq 0$

$$\max_{Q: D_c(Q, P_0) \leq \rho} \mathbb{E}_Q[\ell(\theta_{\mathrm{WRM}}; X, Y)] \leq \gamma\rho + \mathbb{E}_{\widehat{P}_n}[\phi_\gamma(\theta_{\mathrm{WRM}}; X, Y)] + C\frac{\mathfrak{Comp}_n}{\sqrt{n}}$$

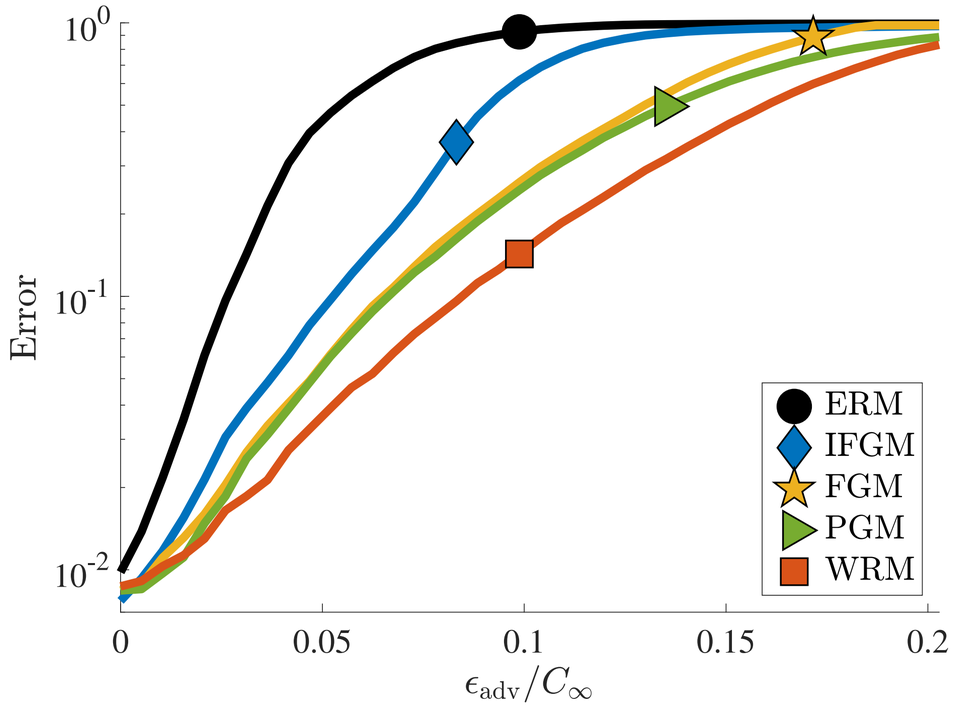
MNIST digit classification

MNIST classification

- Compare our method (WRM) with fast-gradient (FGM), iterated FGM (IFGM), and projected gradient method (PGM)



Test error vs. ϵ_{adv} for PGM $\|\cdot\|_2$ attack

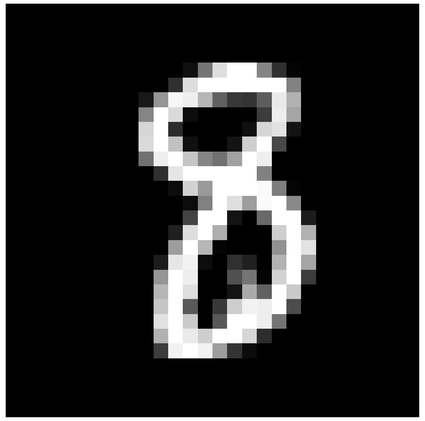


Test error vs. ϵ_{adv} for PGM $\|\cdot\|_\infty$ attack

[Goodfellow et al. 2015, Kurakin et al. 2016, Madry et al. 2017]

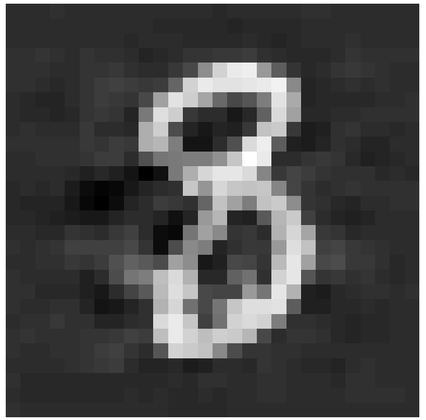
When the model misclassifies

- Minimum perturbation forcing WRM to misclassify is perceptible

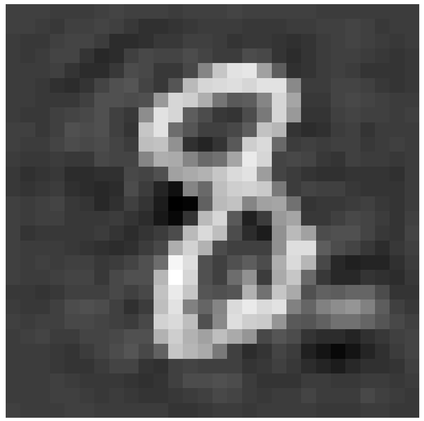


Original

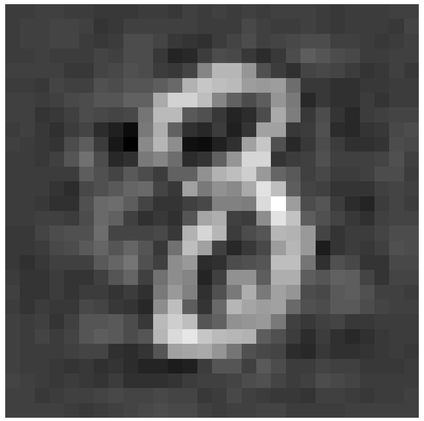
ERM



FGM



IFGM



PGM

WRM

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- What about when $\mathcal{P}$ is large/unknown?

Risk

Find failure modes and quantify the probability of failure

$$\mathbb{P}_0(f(X) < \gamma)$$

- Why is this the right problem?
- How do we solve it quickly?

Balancing safety & performance in high-uncertainty regimes

Sinha*, O'Kelly*, Zheng*, Mangharam, Duchi, Tedrake. ICML 2020.

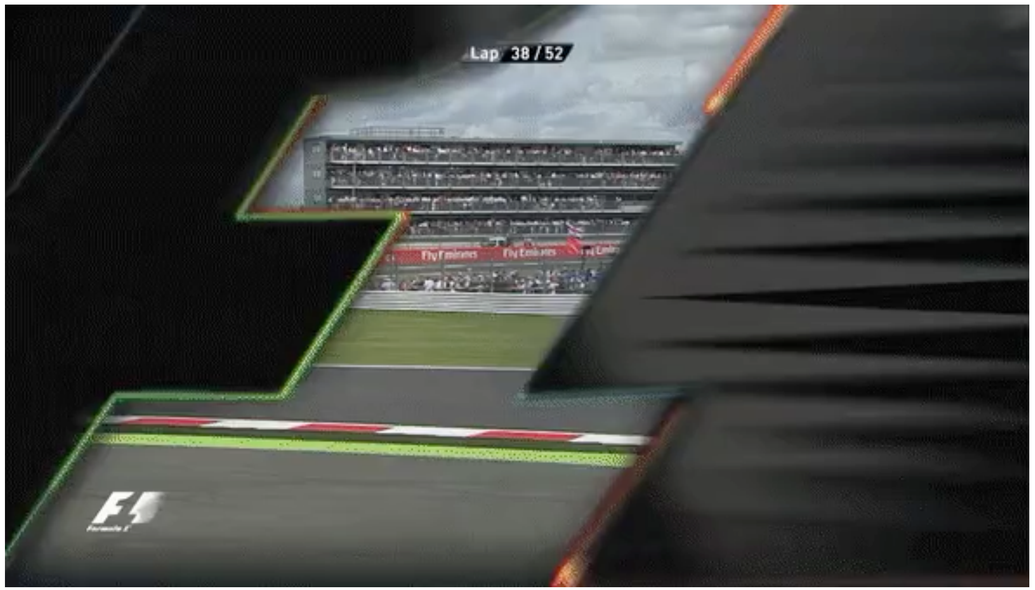




Balancing safety & performance in high-uncertainty regimes

- When $\mathcal{P}$ is large/unknown, balance is critical:
  - Conservativeness leads to poor performance
  - Aggressiveness is dangerous

- Autonomous racing is an extreme limit of autonomous driving
  - Strategies are secret
  - Crashing is expensive/dangerous (and makes winning hard)



Robust reinforcement learning

$$\text{minimize} \sum_t \lambda^t \mathbb{E}[c(o(t))]$$

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- State-action transition probabilities P_{sa}, observations o, discount factor λ, cost c

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- Overall idea: learn a useful parametrization for $\mathcal{P}$ and then proceed as before
  - Offline population synthesis: self-play to learn $\mathcal{P}$, a population of good racers
  - Online robust planning: robust belief-space planning against an opponent

Related work

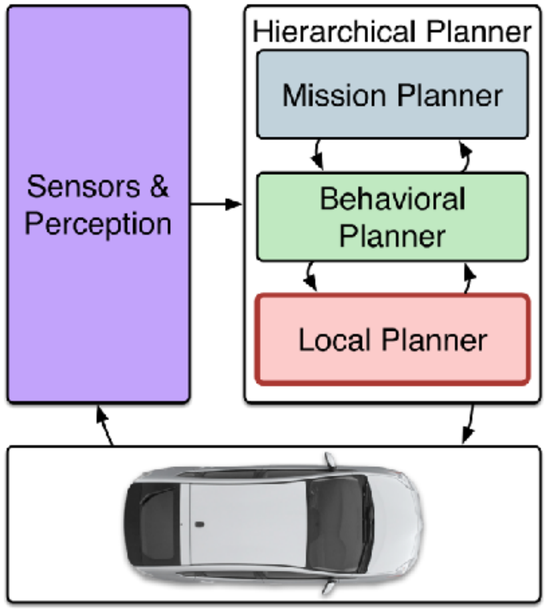
- Robust RL/control
  - Robust MDP [Nilim & El Ghaoui 2005]
  - POMDP [Kaelbling et al. 1998]
  - Adversarial RL [Mandlekar et al. 2017, Pinto et al. 2017]

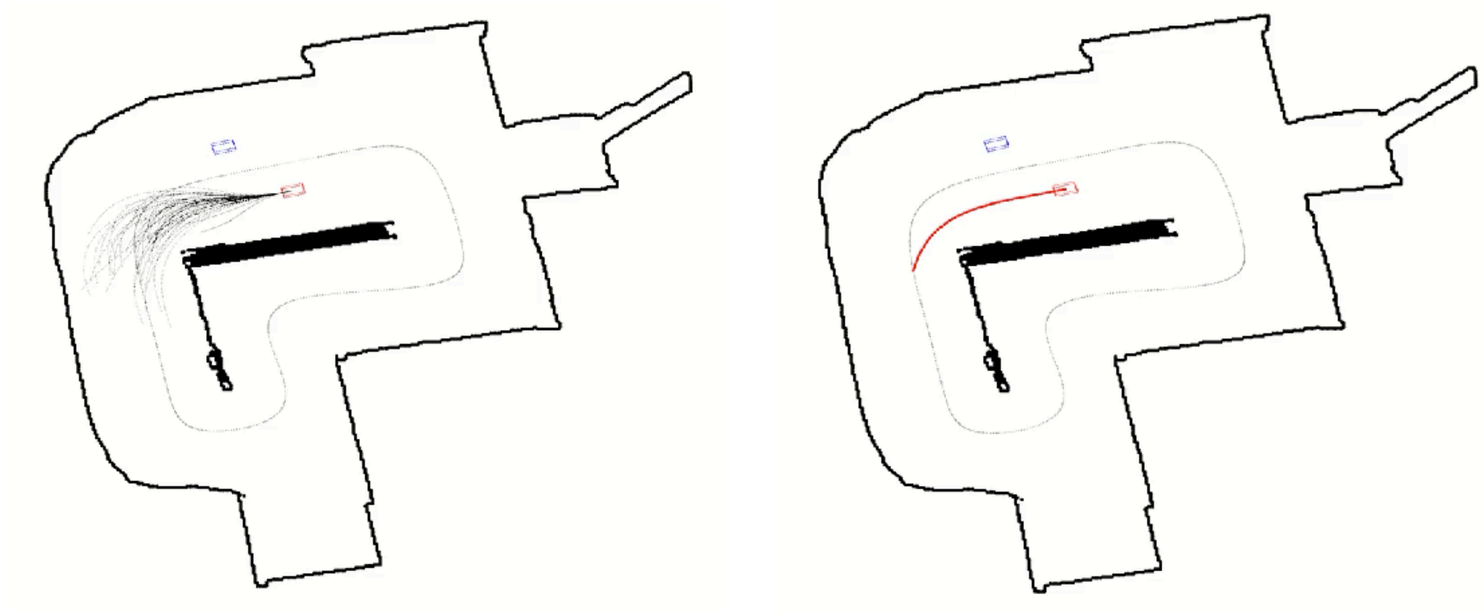
- Belief-space planning [Van Den Berg et al. 2011, Galceran et al. 2015, Kochenderfer 2015]

- DRO [Namkoong & Duchi 2017]

Offline population synthesis

Goal: generate a diverse set of competitive agent behaviors





θ x

Policy parametrization

- Goal generator: neural net (IAF) weights θ
- Goal evaluator: non-differentiable cost weights x

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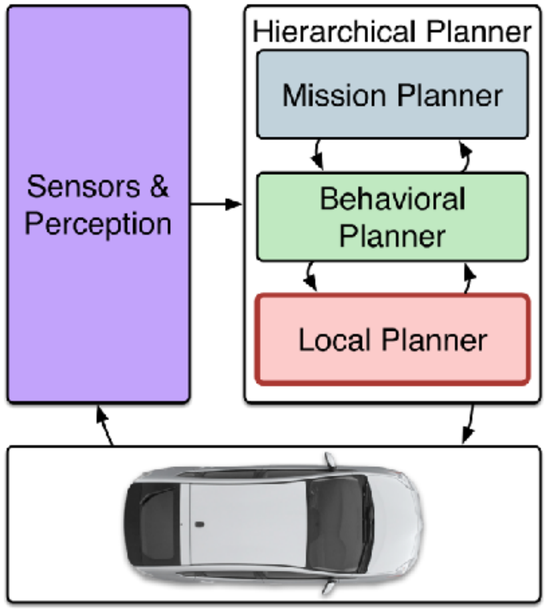

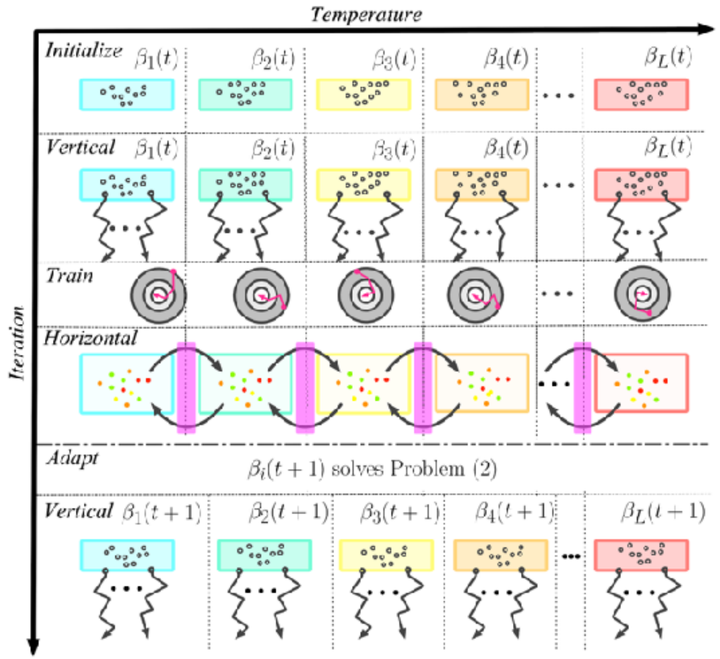

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- Employs self-play to generate competitive agents

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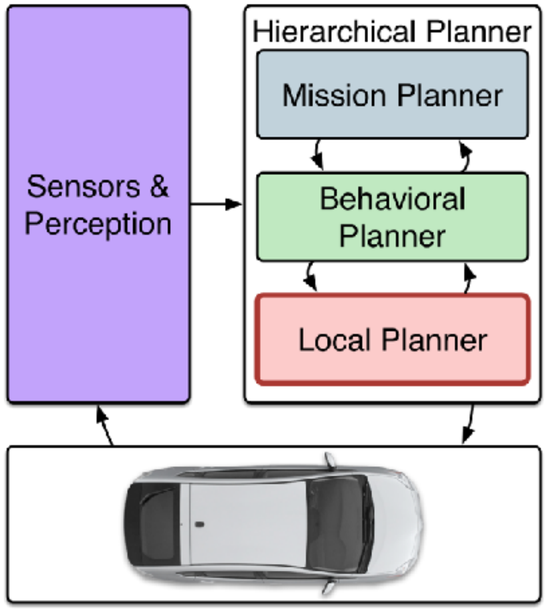

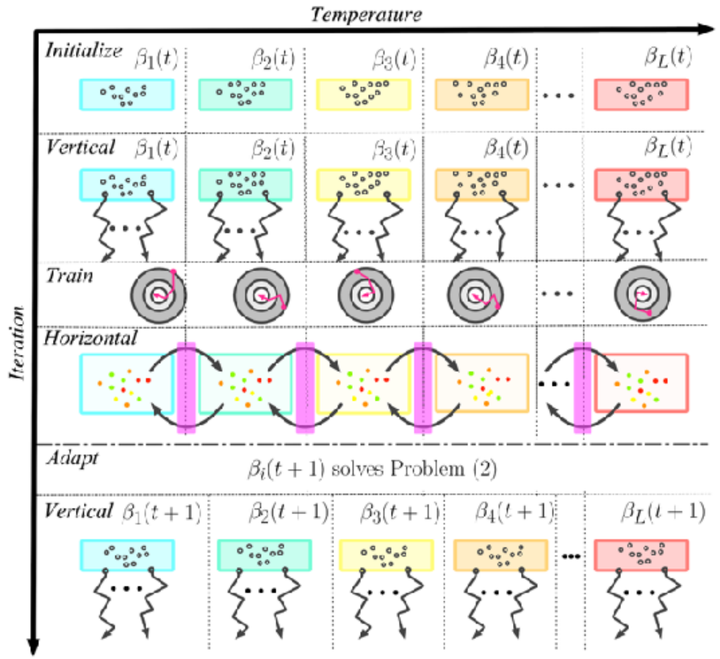

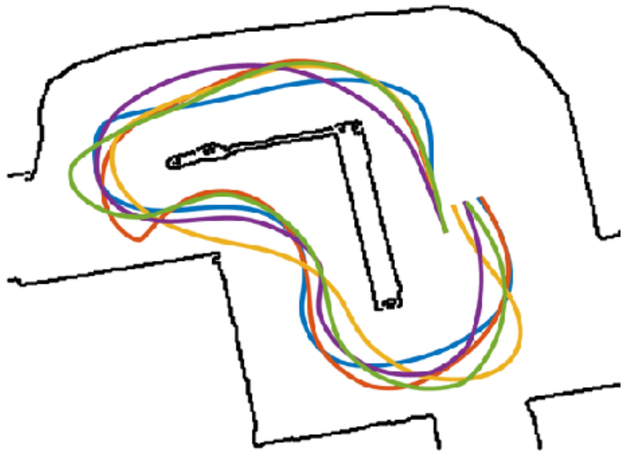

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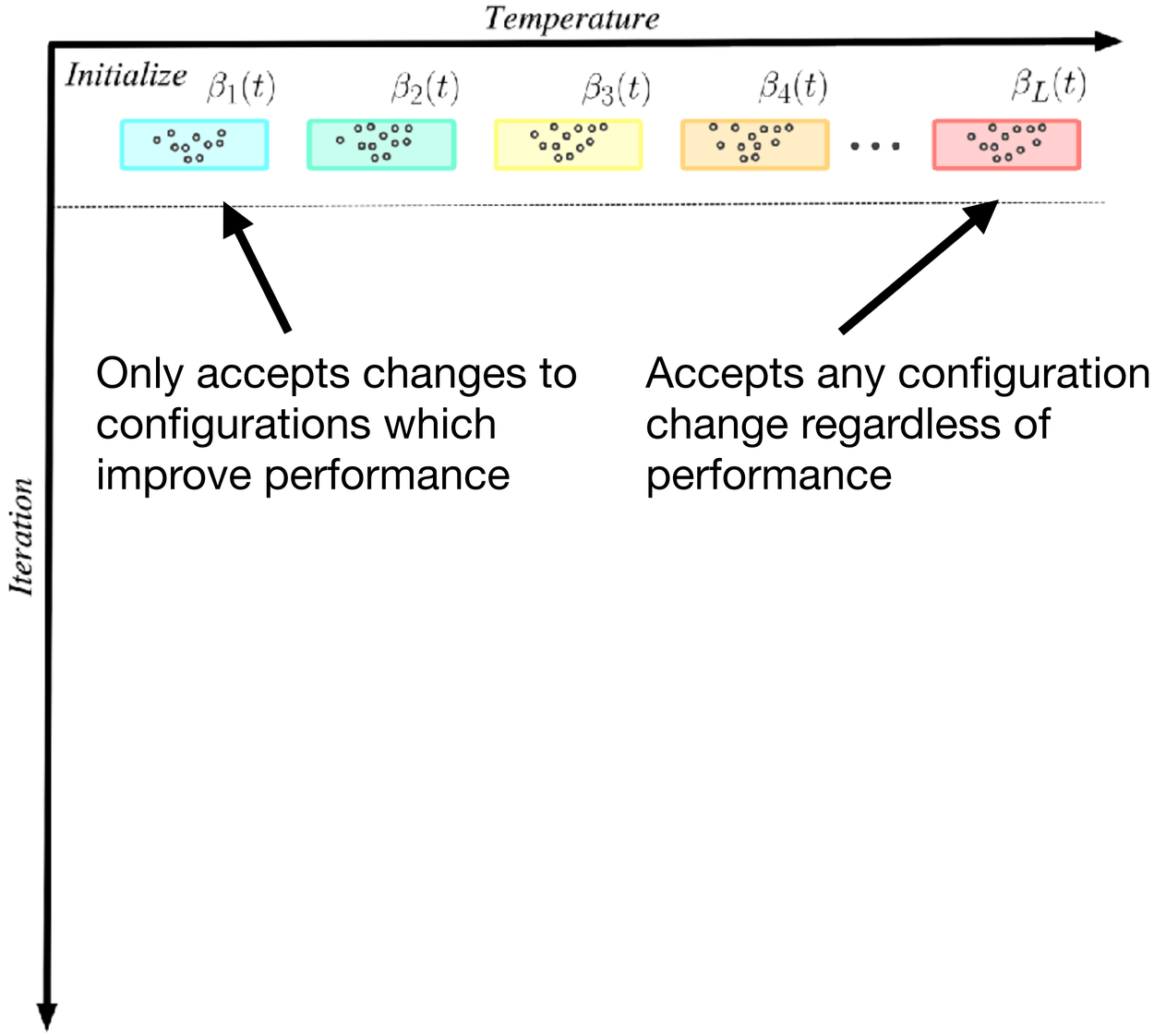
Diverse population of agents

- Described by their parameters (x, θ)

Step 1: initialize populations

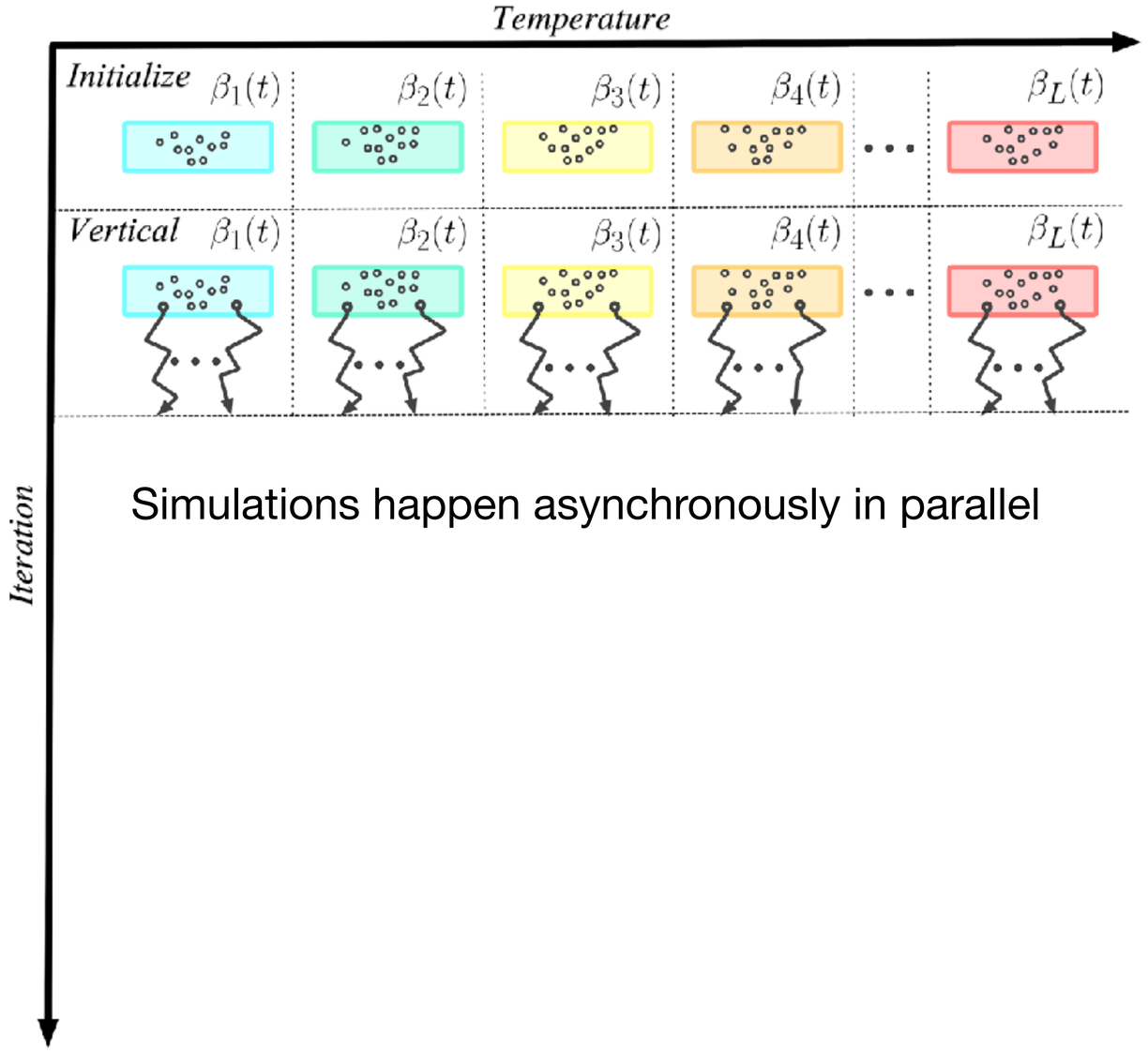
- Builds upon parallel tempering [Marinari & Parisi 1992]

- Initialize several “baths” of configurations (x, θ)



Step 2: self-play (vertical MCMC)

- Explore new proposals for x
- Evaluate each proposal by a race between the old and new configurations



Simulations happen asynchronously in parallel

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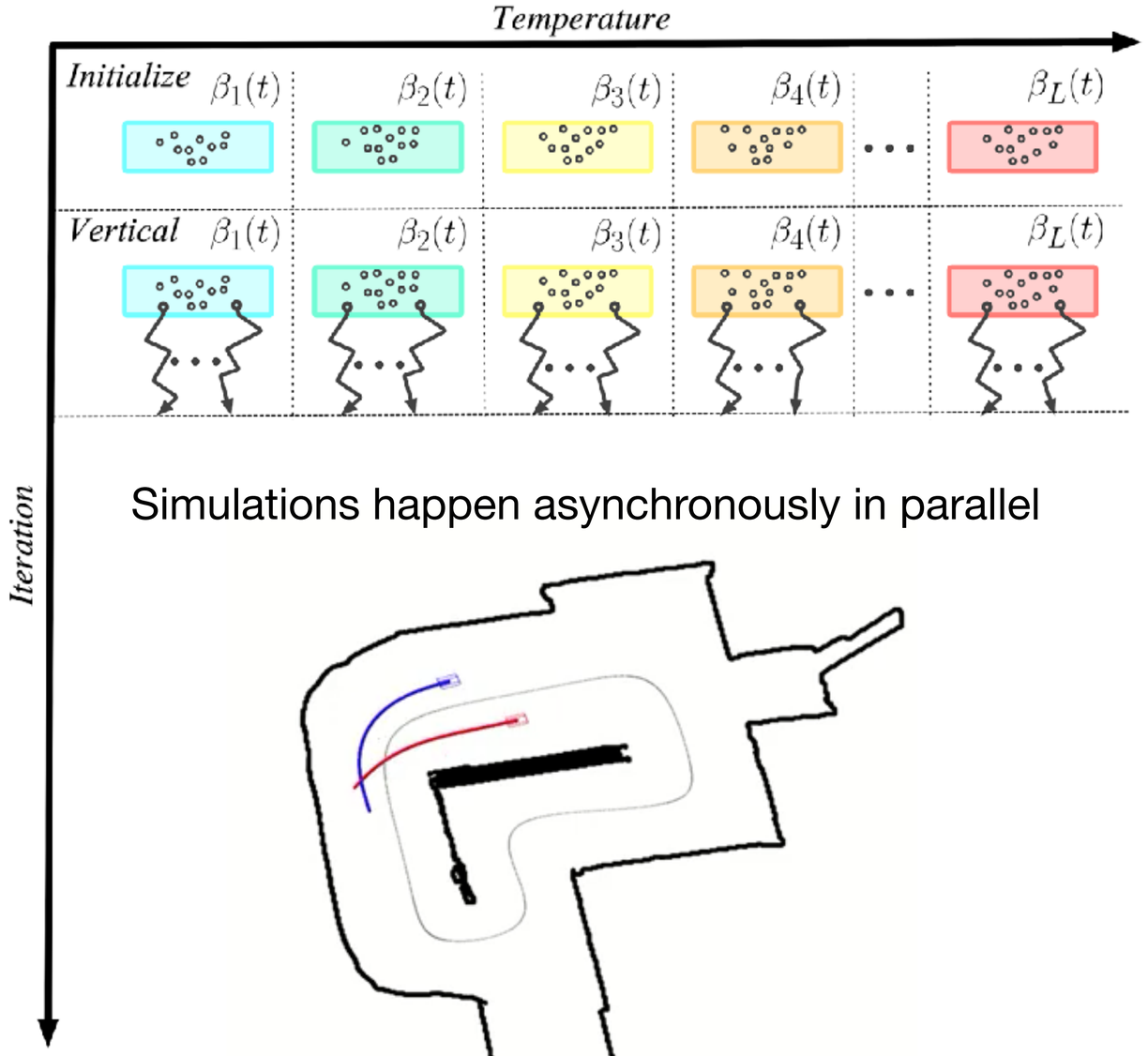

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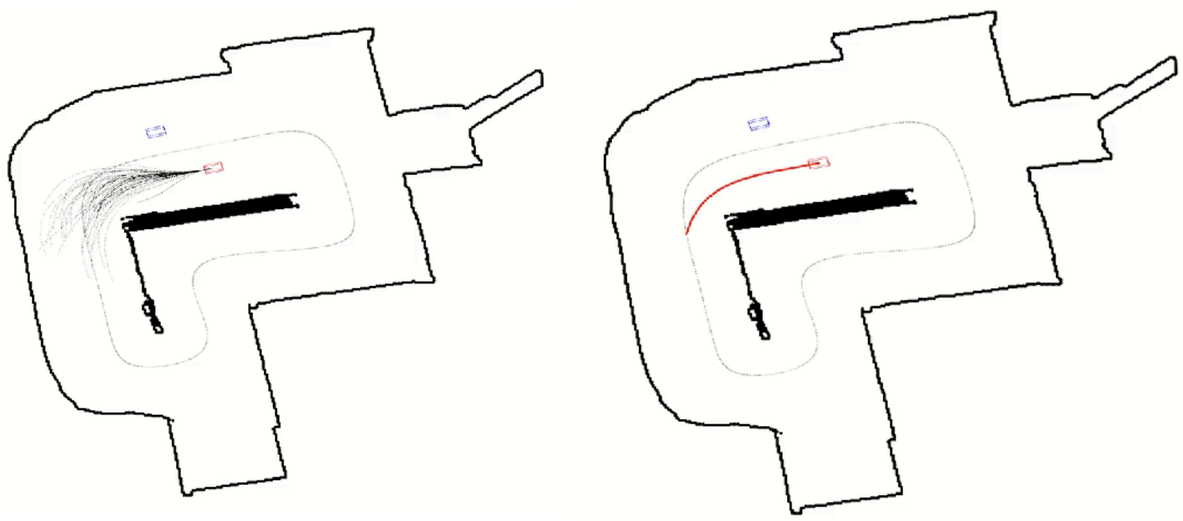
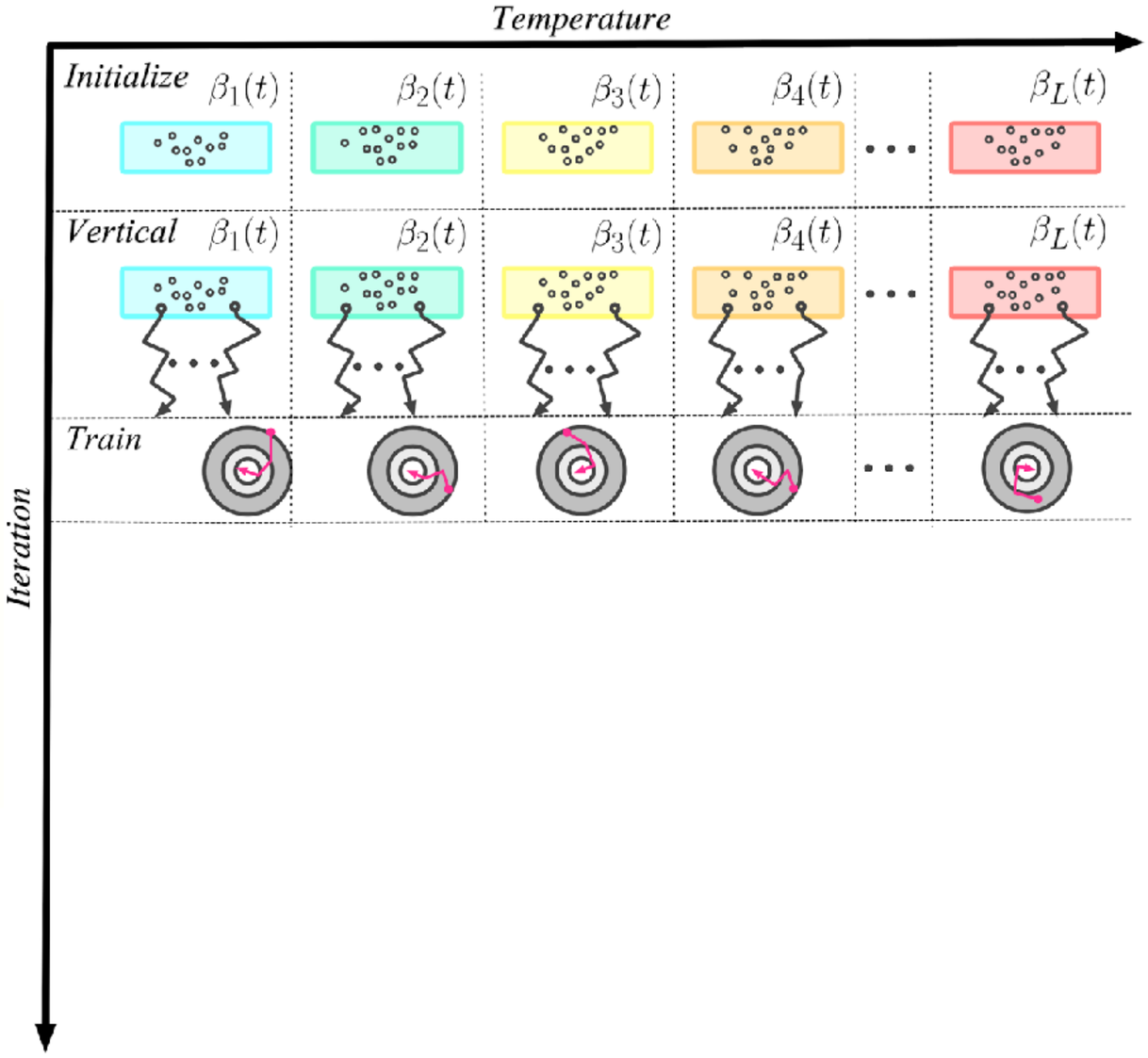
Step 3: differentiable parameter update

- Optimize θ (neural network weights)

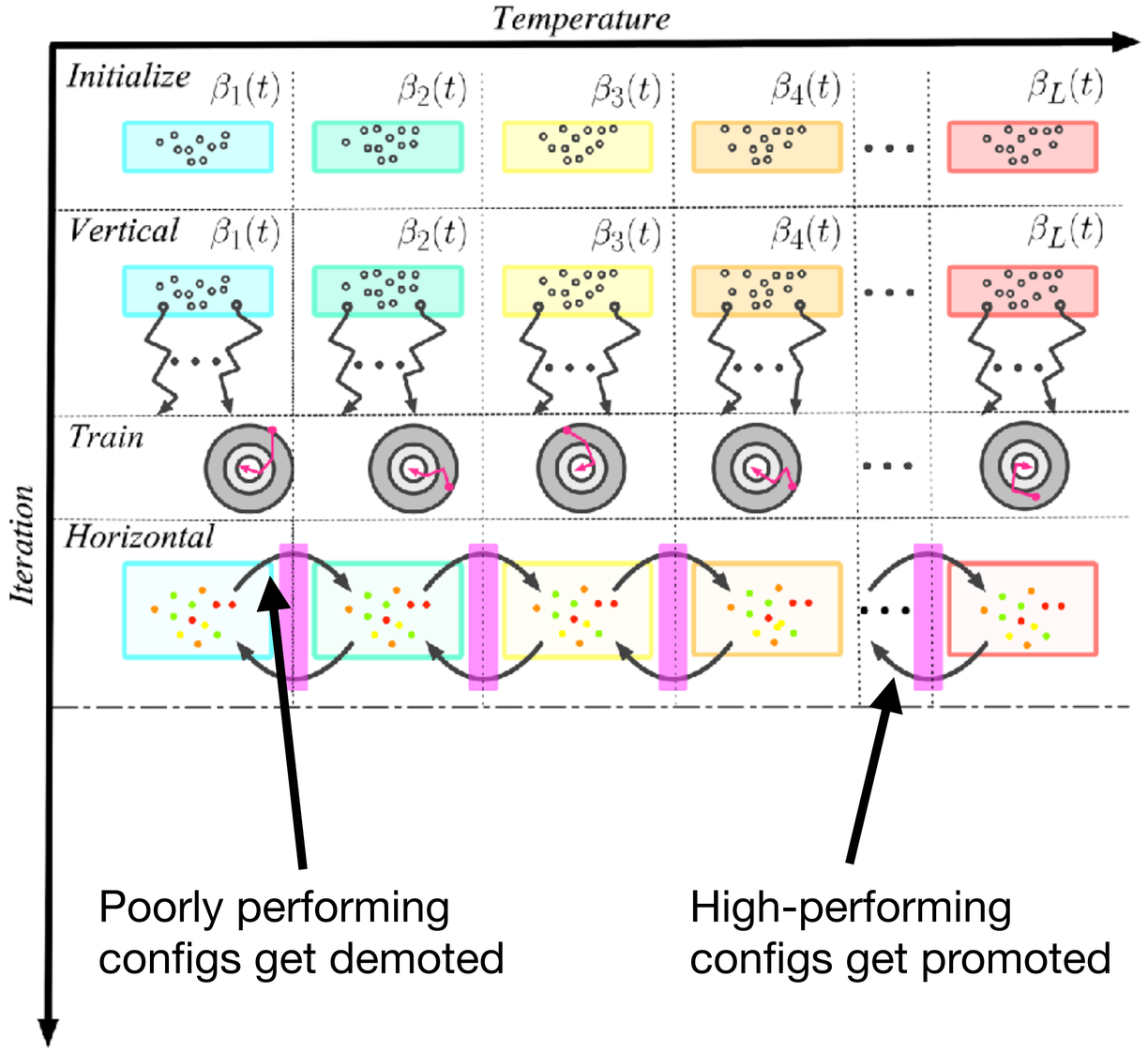




Step 4: configuration swaps (horizontal MCMC)

- Propose random swaps of configurations between adjacent baths

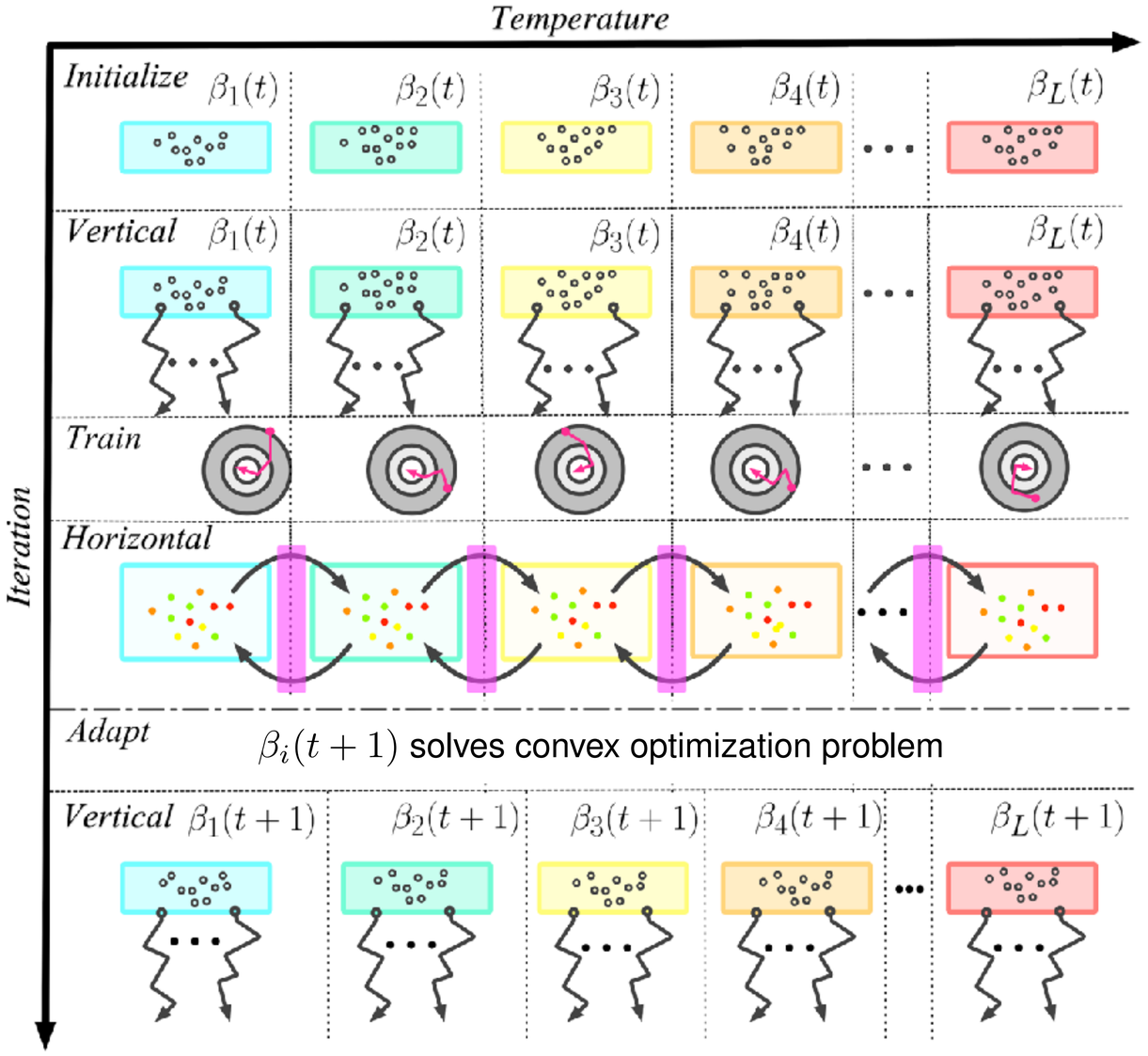
- Efficient way to encourage mixing because no new simulations needed



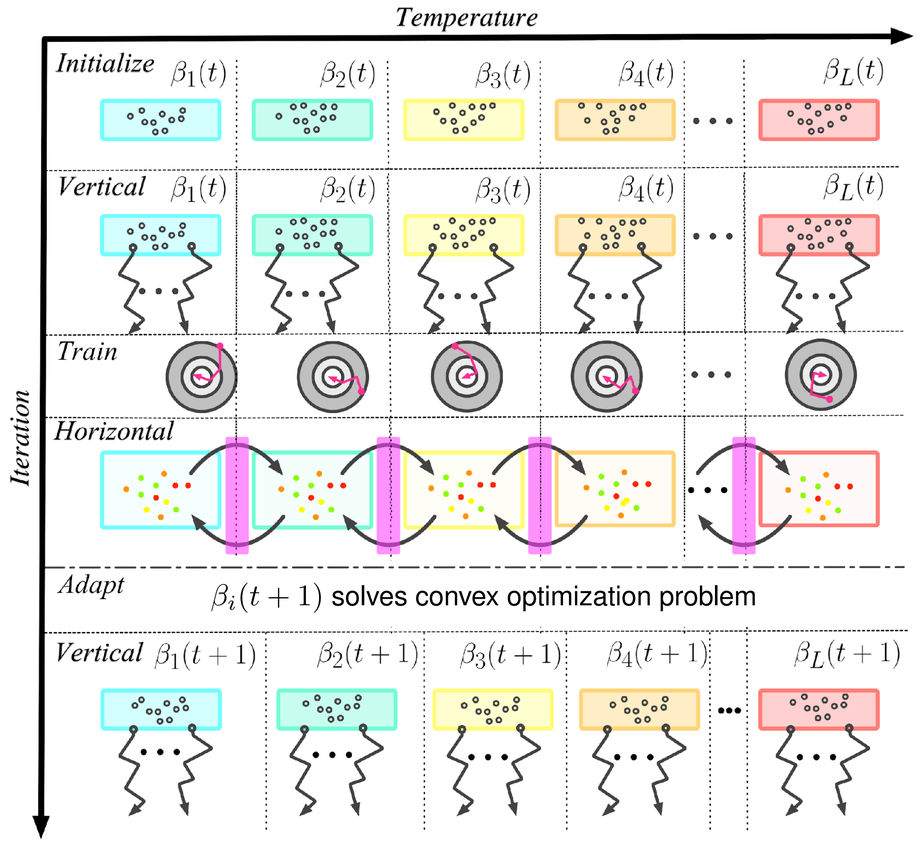
Step 5: update temperatures (annealing)

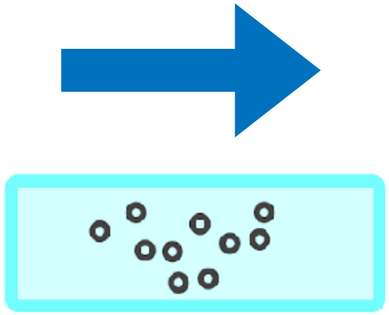
- Adaptive annealing scheme: adjust temperatures by annealing swap-acceptance probability
  - Convex optimization problem

- Crucial in our setting because we don't have any prior knowledge of good race times (no priors for good temperatures)

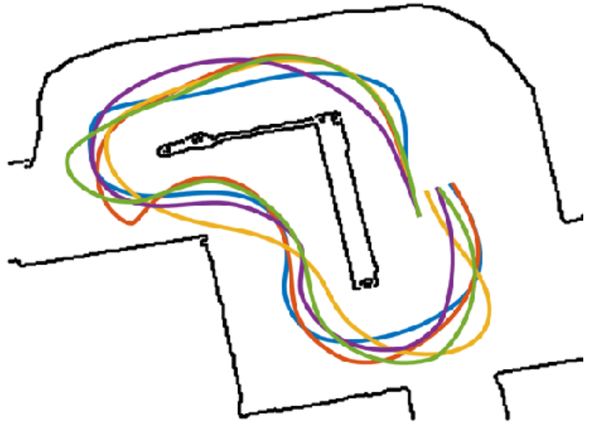


End result: diverse population of opponent prototypes

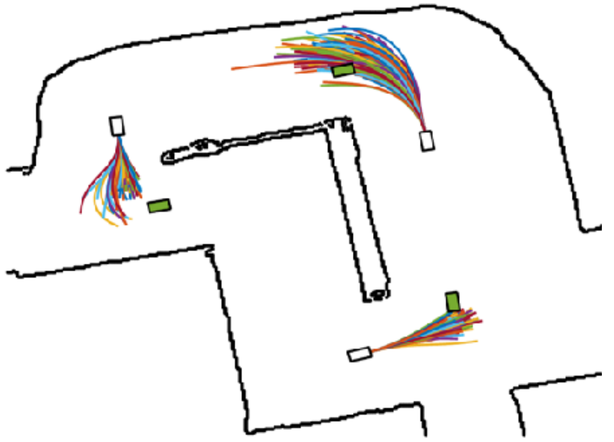




$\beta_1(t)$



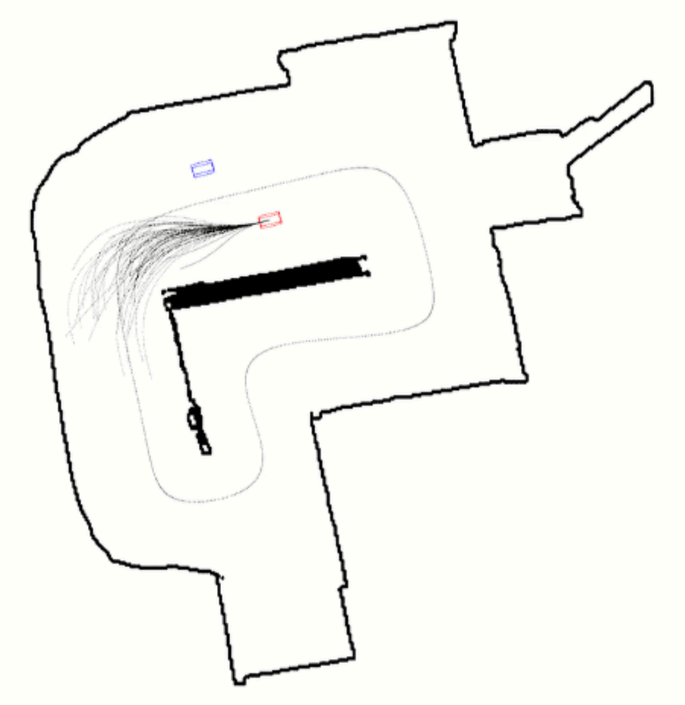
Diversity in isolated laps



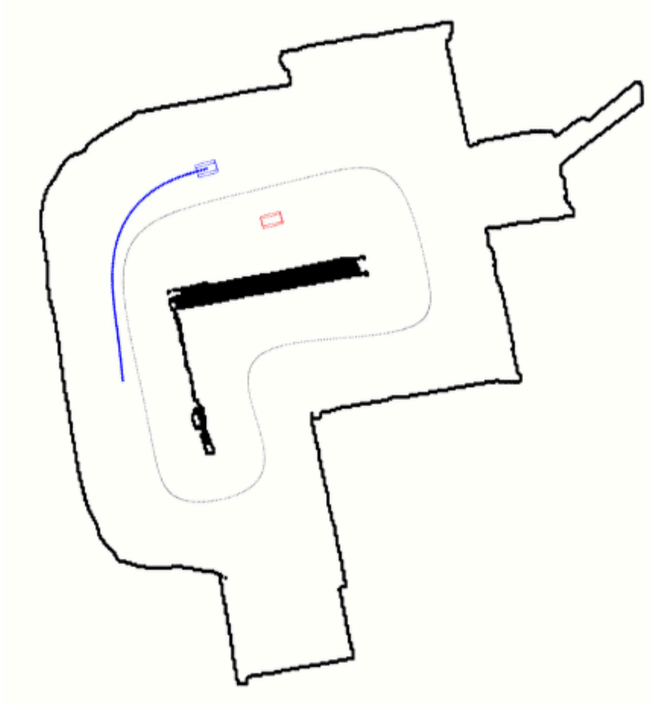
Diversity in maneuvering near an opponent

Distributionally robust planning

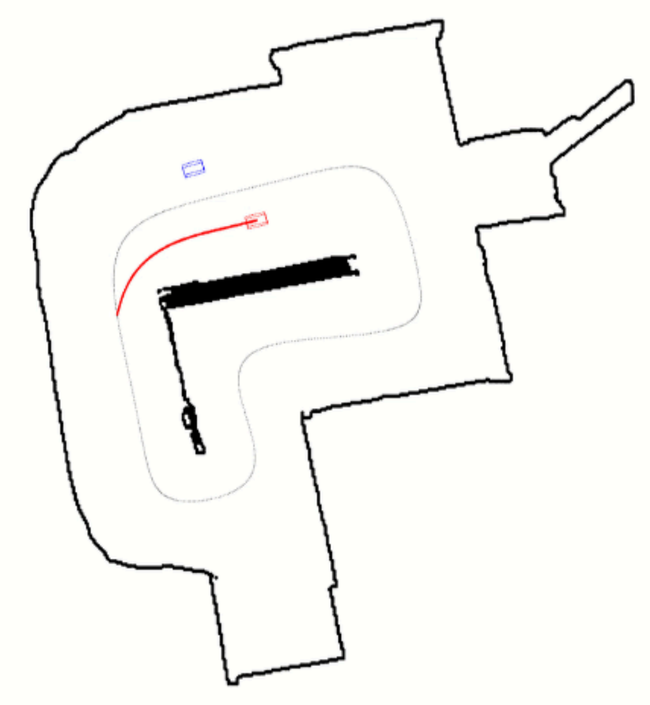
- When racing against an opponent, we maintain a belief vector $w(t)$ of their behavior over the learned population of prototypes
- $\mathcal{P}$ is an uncertainty ball around this belief (χ^2-divergence)



Draw candidate goal



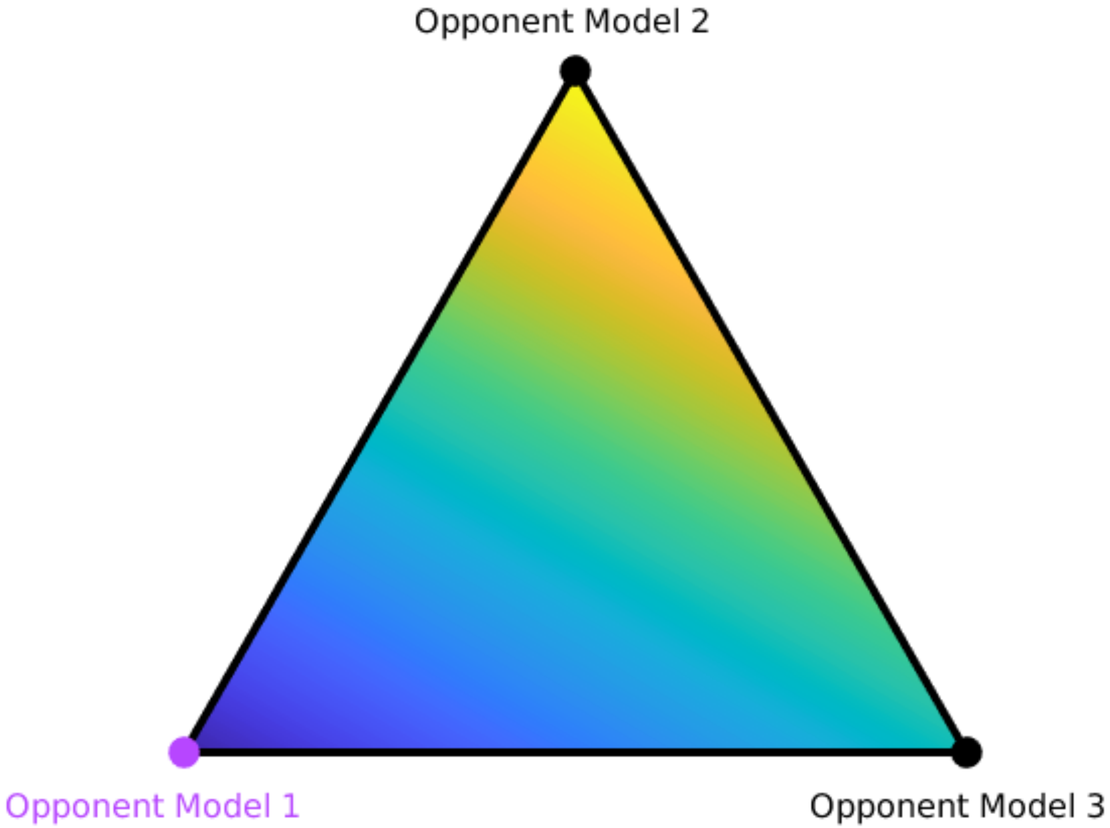
Predict opponent behavior



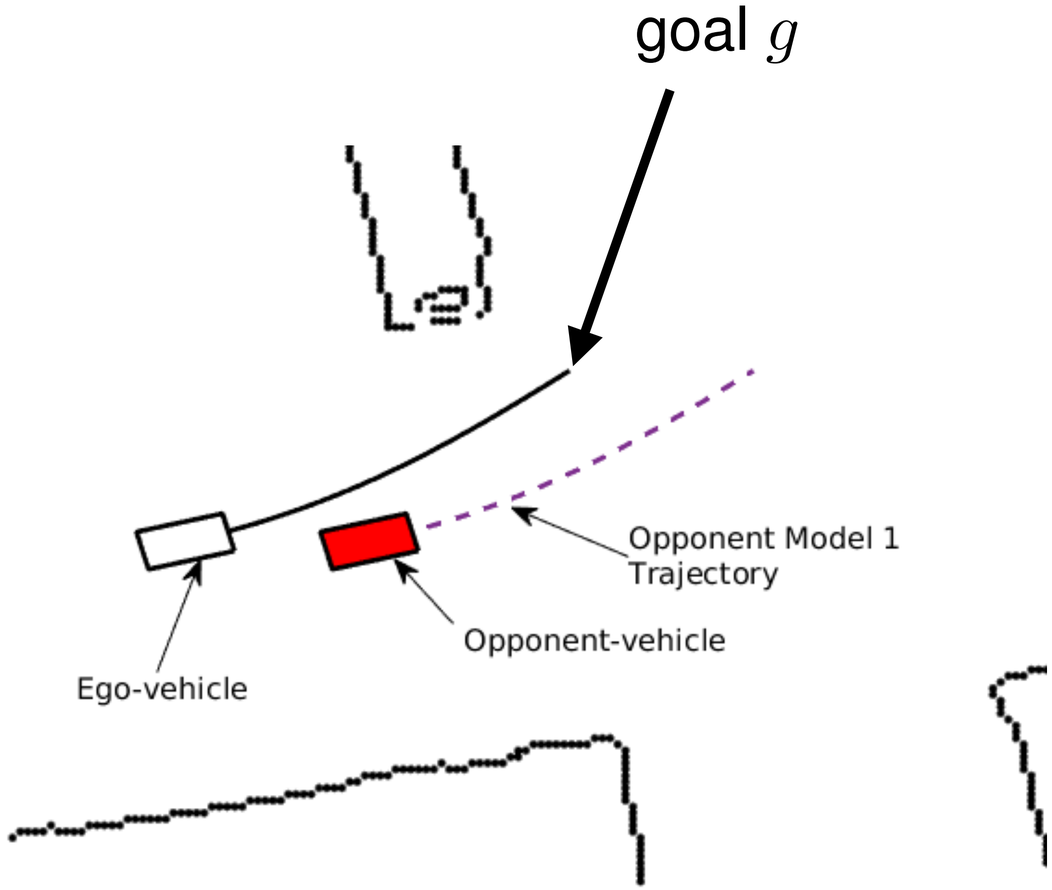
Choose goal

Distributionally robust planning

$$c_1(t;g) := \sum_{s>t} \lambda^{s-t} \mathbb{E}[c(o(s);g)]$$

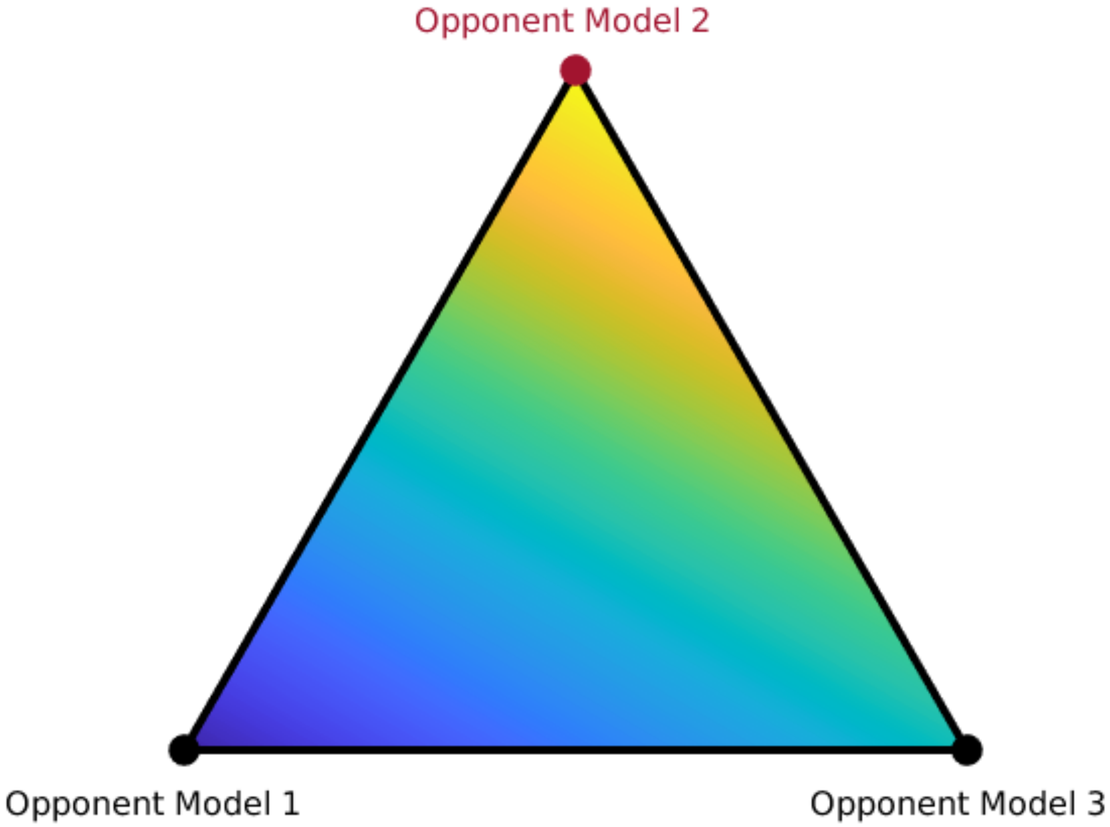


Belief space

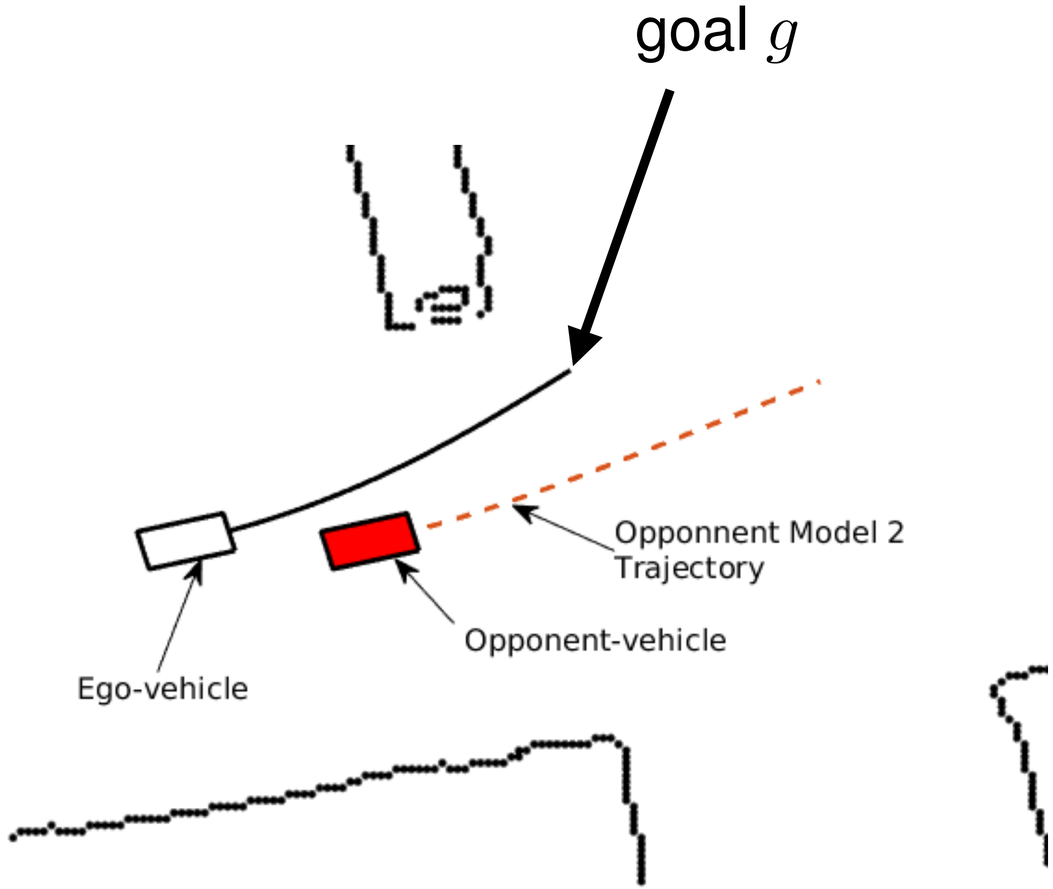


Distributionally robust planning

$$c_2(t; g) := \sum_{s>t} \lambda^{s-t} \mathbb{E}[c(o(s); g)]$$

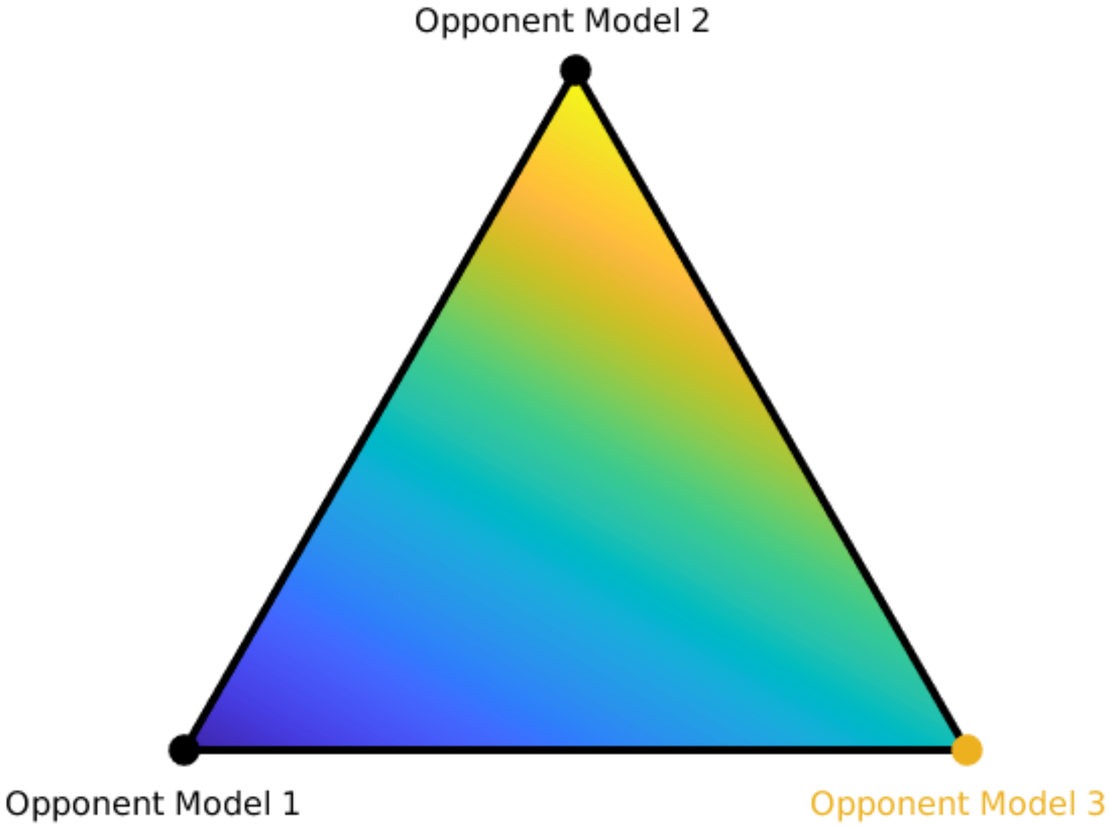


Belief space

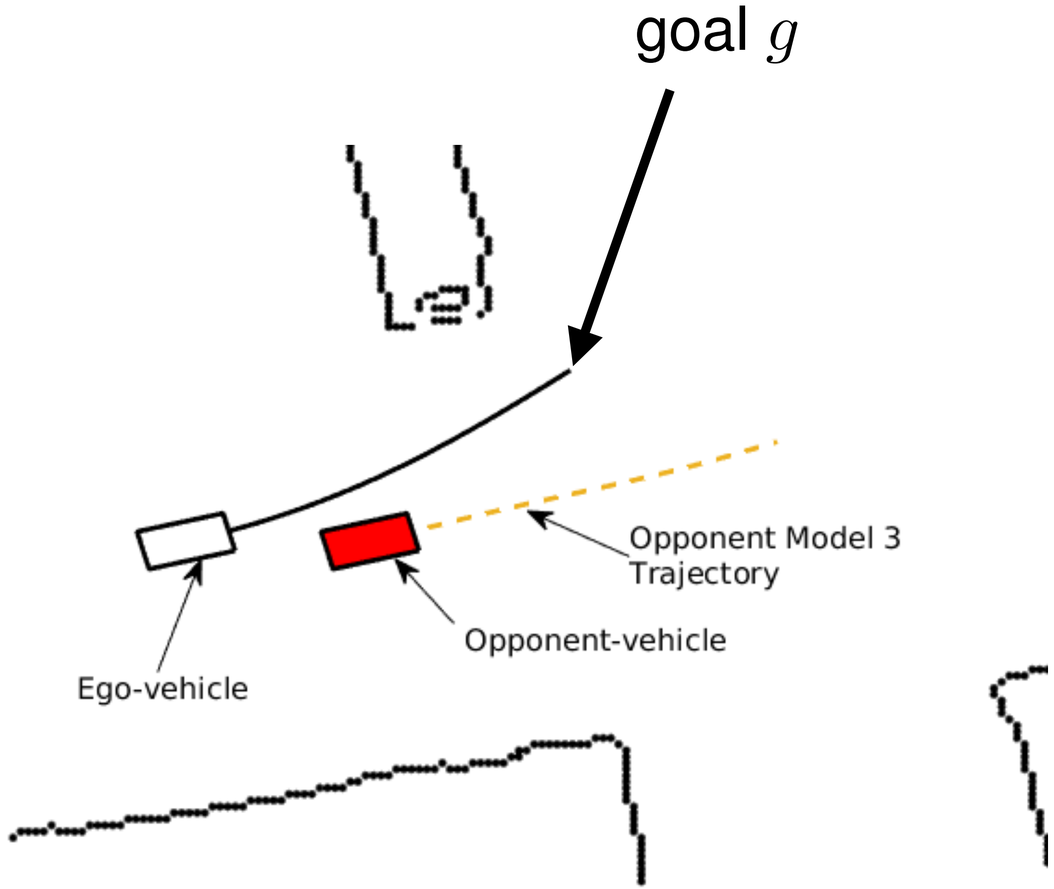


Distributionally robust planning

$$c_3(t;g) := \sum_{s>t} \lambda^{s-t} \mathbb{E}[c(o(s);g)]$$

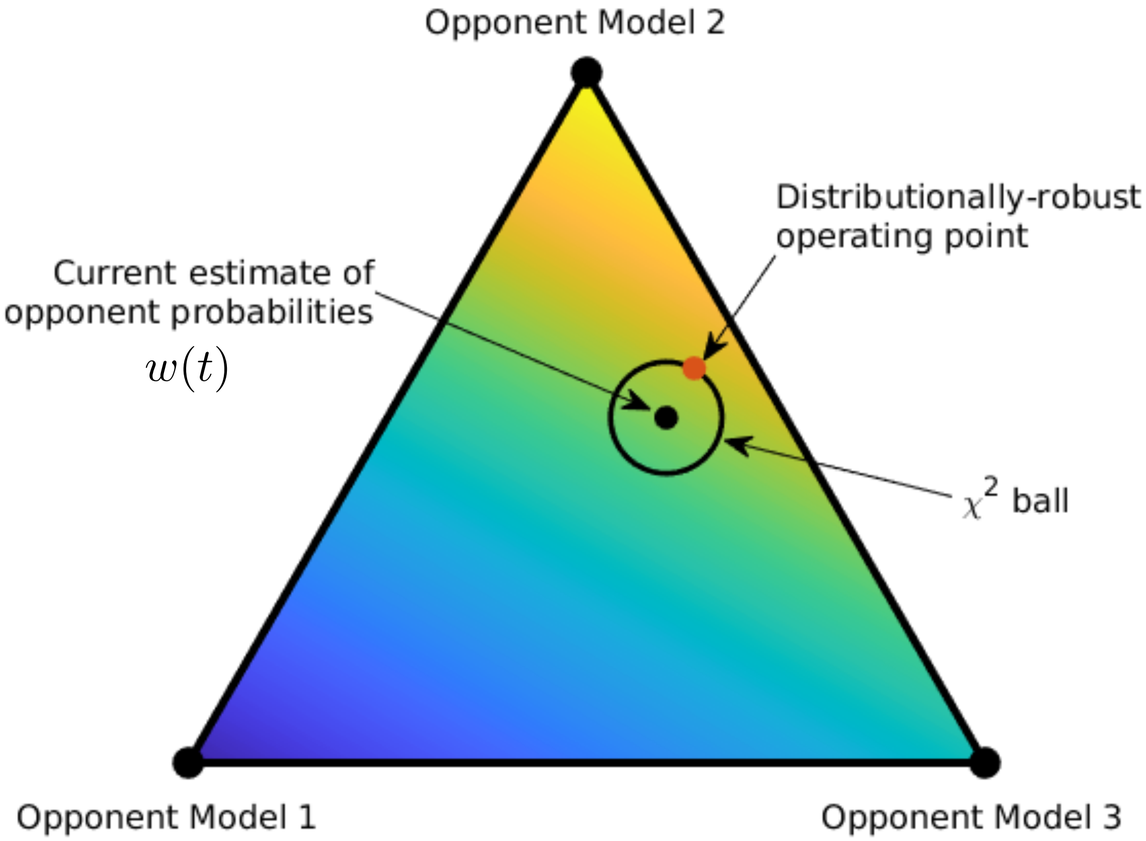


Belief space

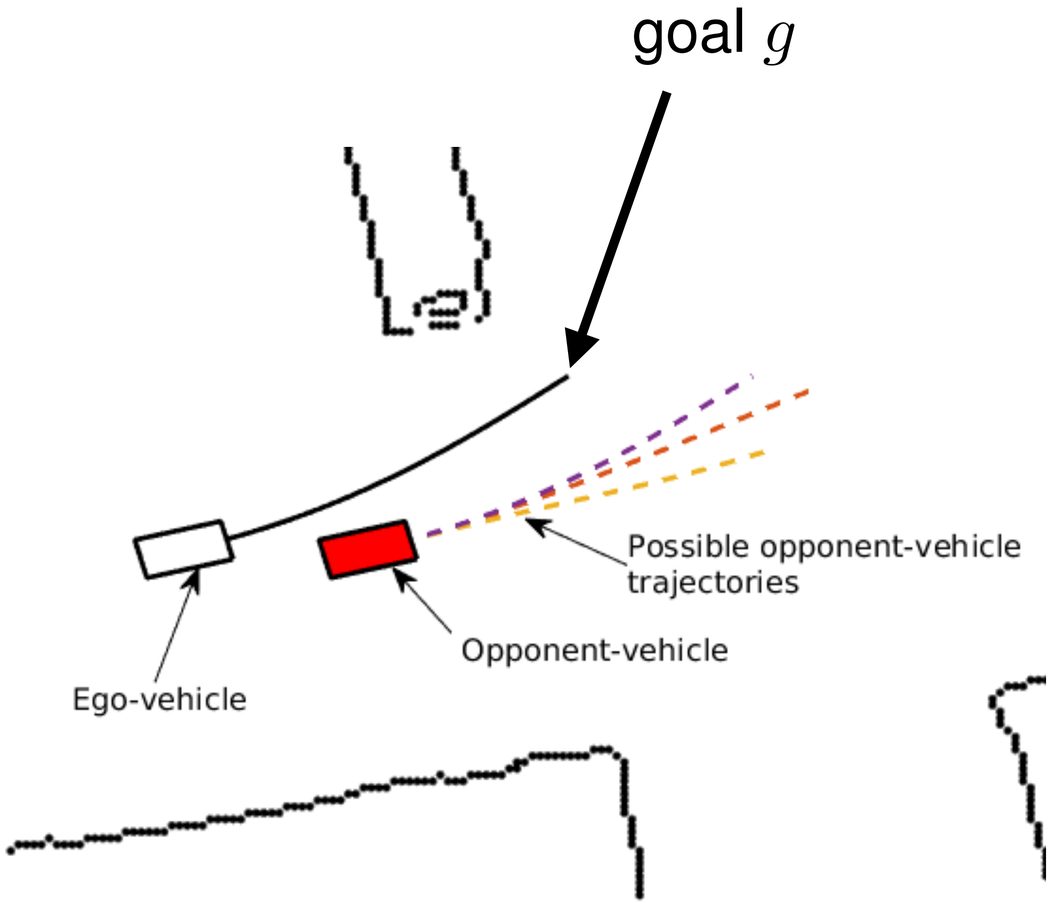


Distributionally robust planning

$$\max_{q:\sum_i w_i(\frac{q_i}{w_i})^2 \leq \rho+1} \sum_i q_i c_i(t;g)$$

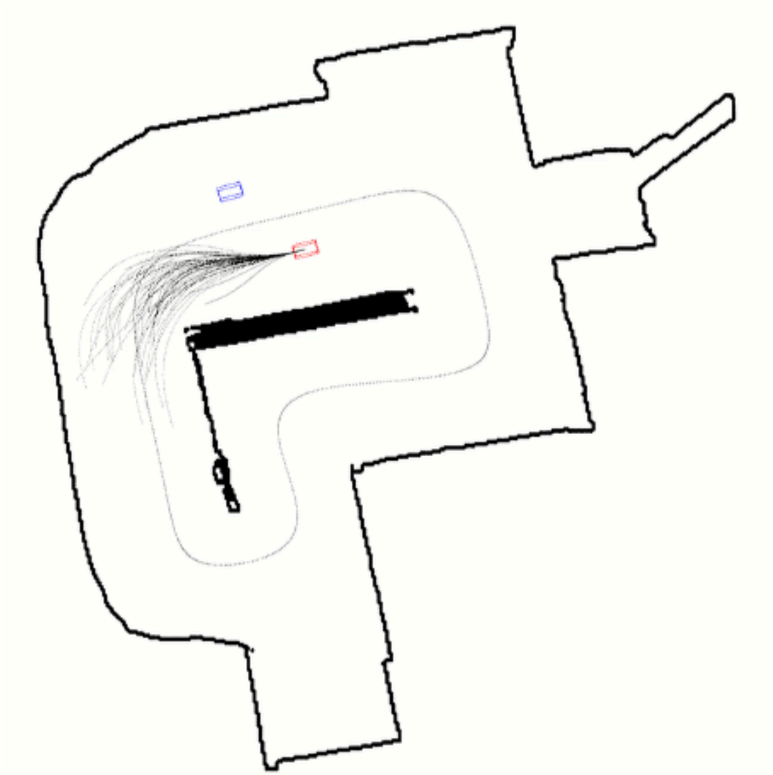


Belief space

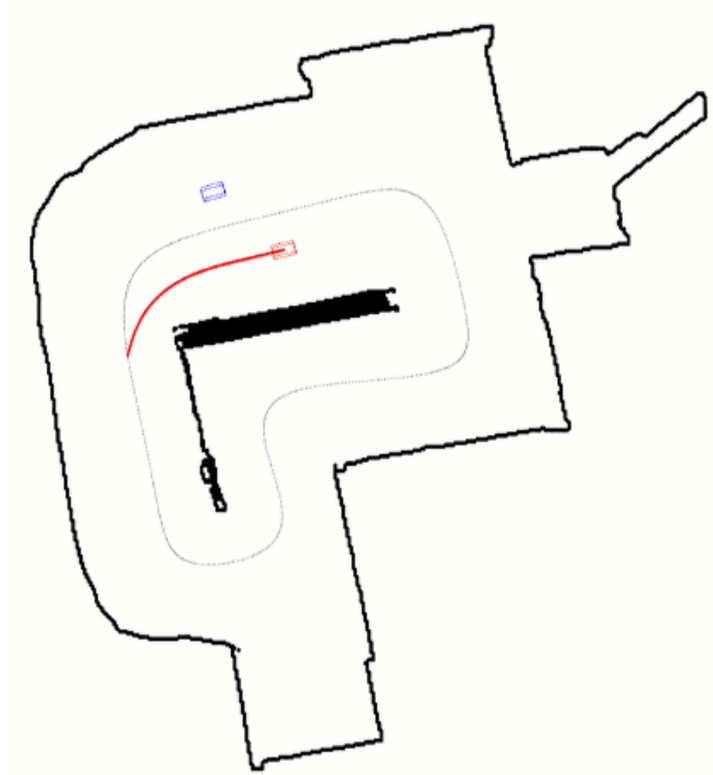


Distributionally robust planning

Repeat this for every motion planning goal and select the goal with the lowest robust cost

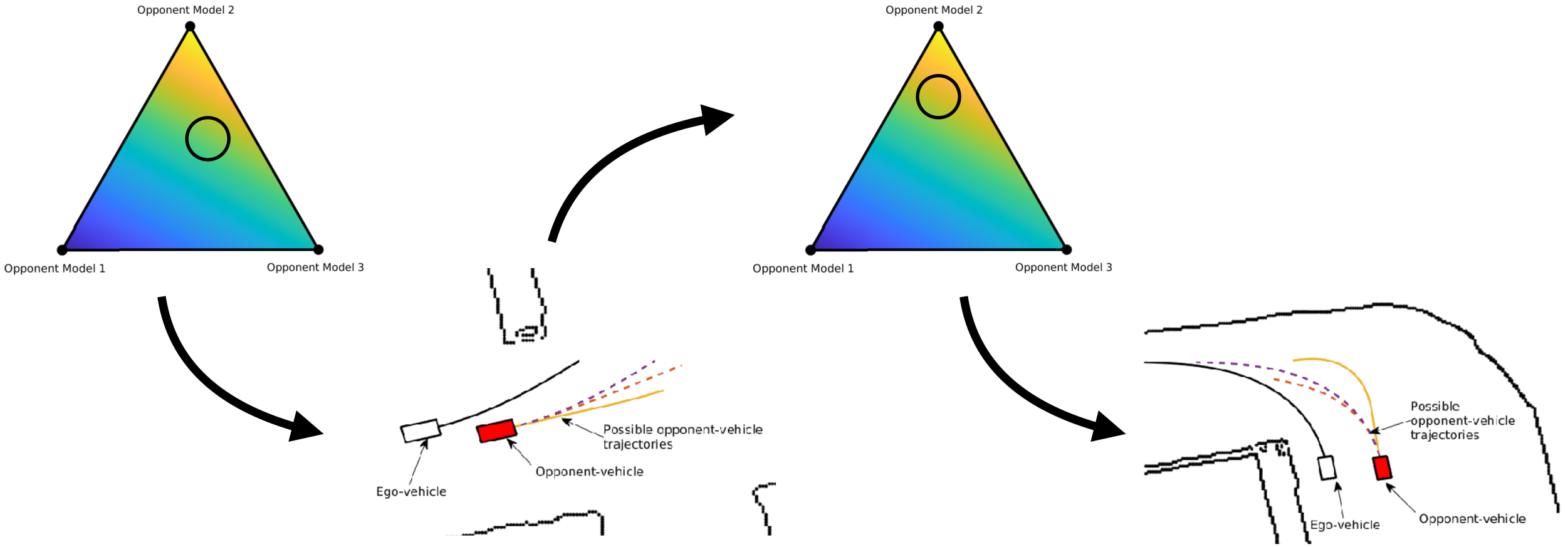


$$\max_{q:\sum_i w_i(\frac{q_i}{w_i})^2 \leq \rho+1} \sum_i q_i c_i(t;g)$$

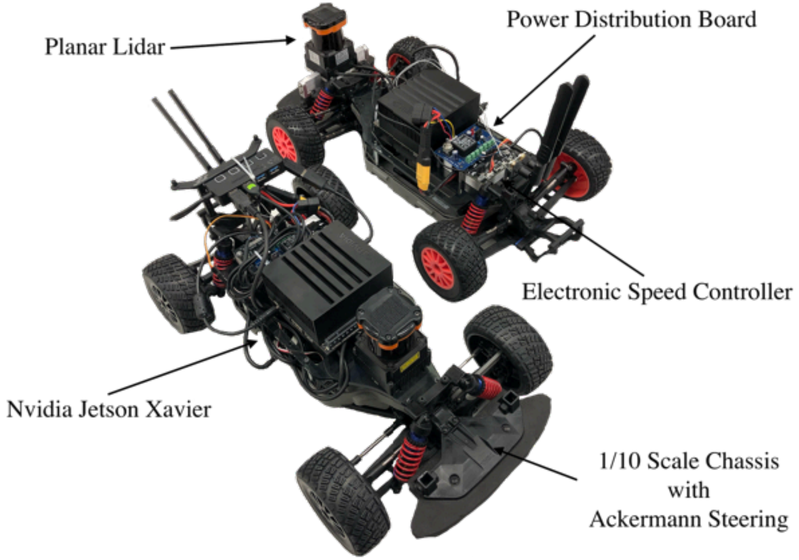


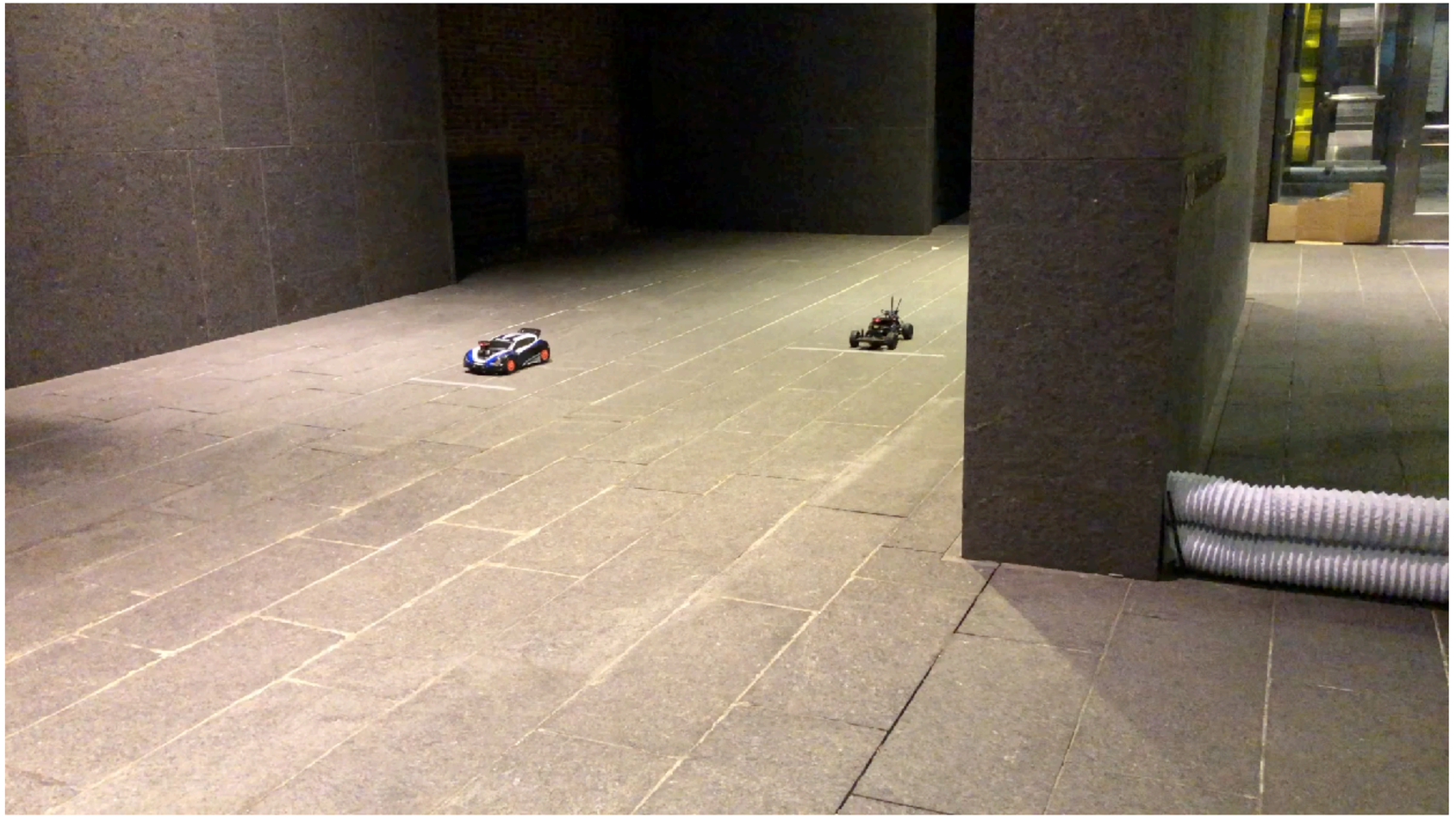
Belief updates (adaptivity)

- Update beliefs using the observed history of the opponent
  - Modified version of EXP3 [Auer et al. 2002]



Real-world experiments





Balancing safety and performance

Larger uncertainty sets (larger ρ) increase safety but decrease performance

Agent	% of iTTC values < 0.5s
$\rho/N_w = 0.001$	$\mathbf{7.86 \pm 0.90}$
$\rho/N_w = 0.025$	6.46 ± 0.78
$\rho/N_w = 0.2$	4.75 ± 0.65
$\rho/N_w = 0.4$	5.41 ± 0.74
$\rho/N_w = 0.75$	5.50 ± 0.82
$\rho/N_w = 1.0$	$\mathbf{5.76 \pm 0.84}$

Increased safety with ρ

Agent	Win-rate Non-adaptive
$\rho/N_w = 0.001$	$\mathbf{0.593 \pm 0.025}$
$\rho/N_w = 0.025$	0.593 ± 0.025
$\rho/N_w = 0.2$	0.538 ± 0.025
$\rho/N_w = 0.4$	0.503 ± 0.025
$\rho/N_w = 0.75$	0.513 ± 0.025
$\rho/N_w = 1.0$	0.498 ± 0.025

Decreased win-rate with ρ

Balancing safety and performance

Online adaptation regains the performance of aggressive strategies:
Safe when uncertain, aggressive once the opponent is identified

Agent	Win-rate Non-adaptive	Win-rate Adaptive	p-value
$\rho/N_w = 0.001$	$\mathbf{0.593 \pm 0.025}$	0.588 ± 0.025	0.84
$\rho/N_w = 0.025$	0.593 ± 0.025	0.600 ± 0.024	0.77
$\rho/N_w = 0.2$	0.538 ± 0.025	0.588 ± 0.025	0.045
$\rho/N_w = 0.4$	0.503 ± 0.025	0.573 ± 0.025	0.0098
$\rho/N_w = 0.75$	0.513 ± 0.025	0.593 ± 0.025	0.0013
$\rho/N_w = 1.0$	0.498 ± 0.025	$\mathbf{0.590 \pm 0.025}$	0.00024

Key ideas of this talk

Robustness

Build models with guaranteed performance over uncertainty sets

$$\min_{\theta \in \Theta} \max_{Q \in \mathcal{P}} \mathbb{E}_Q[f(\theta; X)]$$

- With small $\mathcal{P}$, can we solve this quickly?
- What about when $\mathcal{P}$ is large/unknown?

Risk

Find failure modes and quantify the probability of failure

$$\mathbb{P}_0(f(X) < \gamma)$$

- Why is this the right problem?
- How do we solve it quickly?

The risk-based framework

O’Kelly*, Sinha*, Namkoong*, Duchi, Tedrake. NeurIPS 2018.
O’Kelly*, Sinha*, Norden*, Namkoong*. NeurIPS ML4H 2018.





The risk-based framework

Tesla Autopilot crashing in highway scenarios

The risk-based framework

- Certify a level of reliability
- Work with a blackbox algorithm

Related work

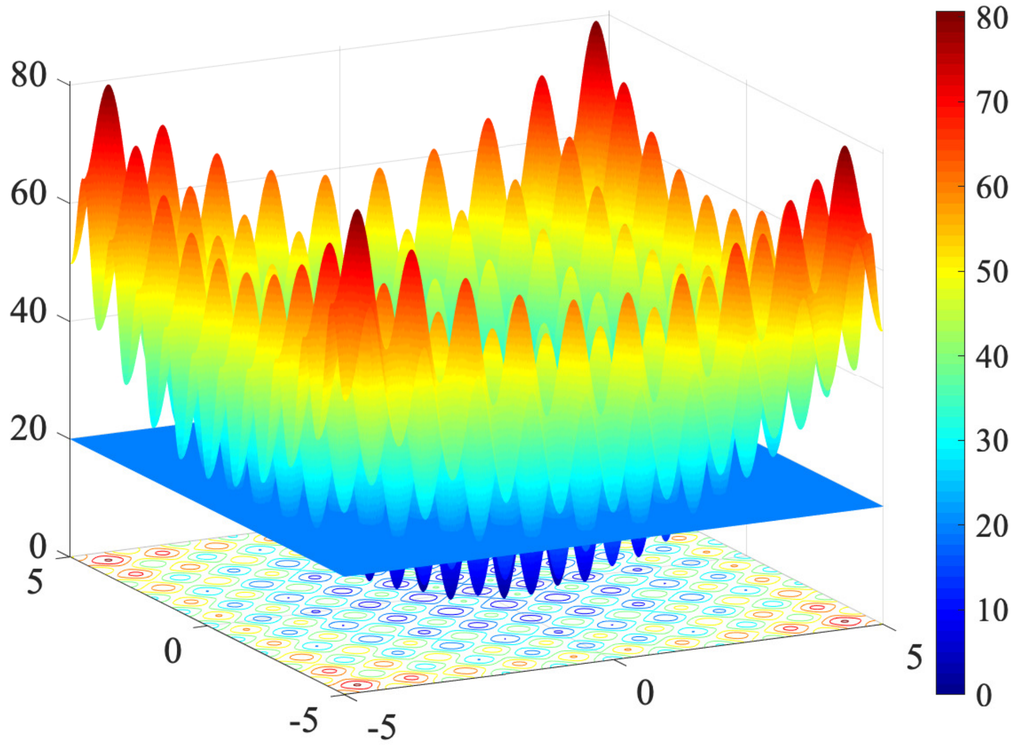
- Formal verification [Kwiatkowska et al. 2011, Althoff and Dolan 2014, Seshia et al. 2015, O'Kelly et al. 2016]
- Falsification [Abbas and Fainekos 2011, Tuncali et al. 2016, DeCastro et al. 2018]
- Probabilistic falsification/Adaptive stress testing [Koren et al. 2018, Lee et al. 2018]

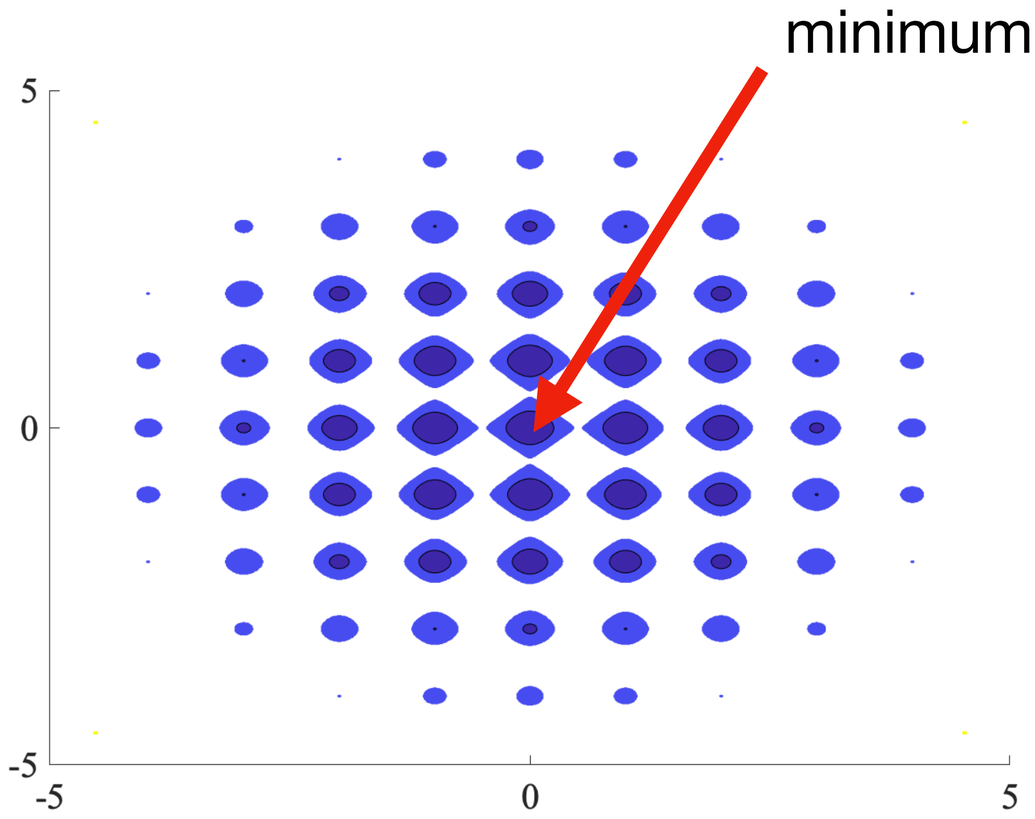
Problems with verification

- A “correctness” specification is subjective
- Intractable
- Requires white-box model

Problems with falsification

- Not designed for coverage



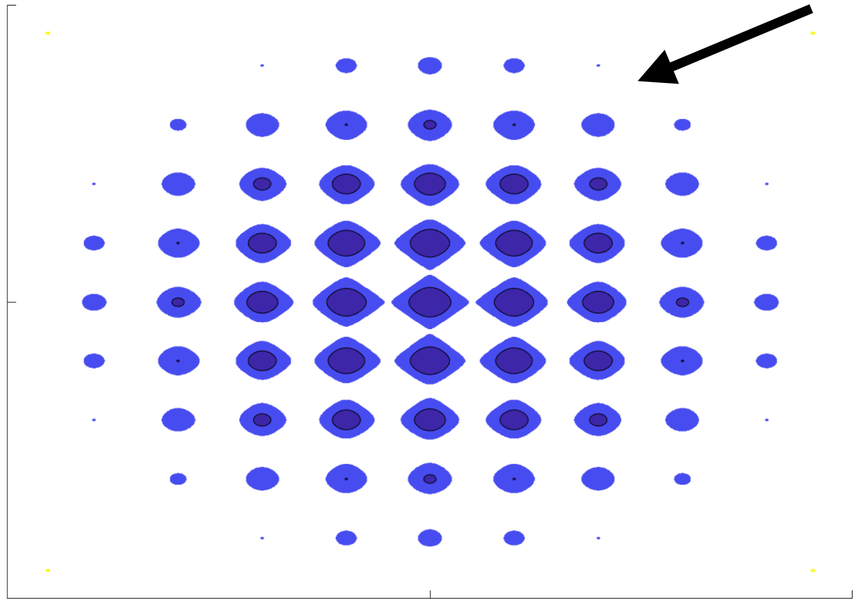


The risk-based framework

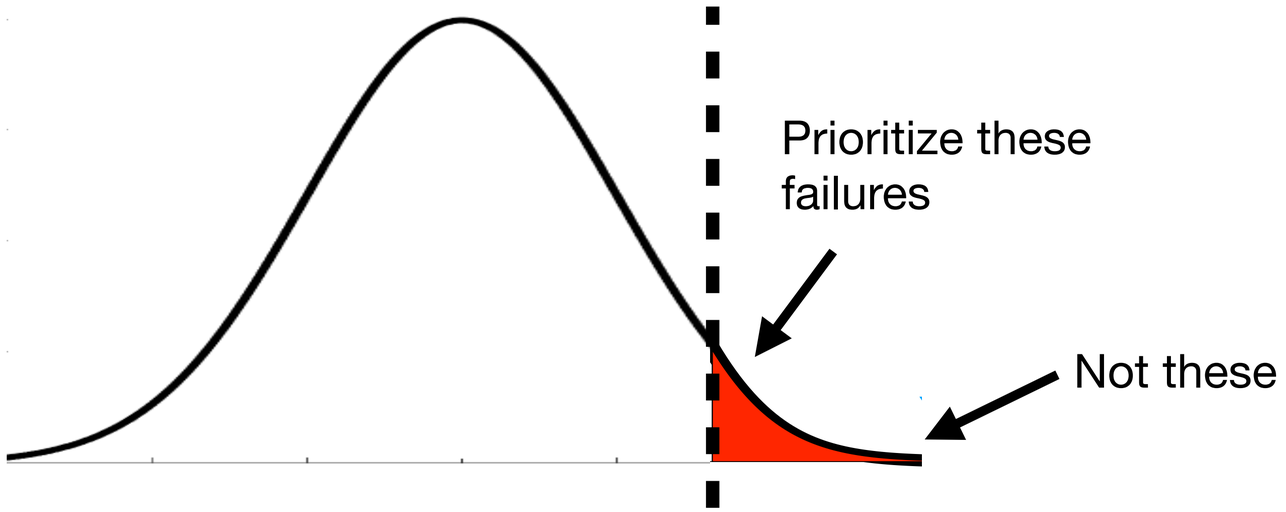
- Goal: probability of dangerous event $p_\gamma := \mathbb{P}_0(f(X) < \gamma)$
- Base distribution of behavior $X \sim P_0$
- Objective function (i.e. safety metric) $f : \mathcal{X} \to \mathbb{R}$

The risk-based framework

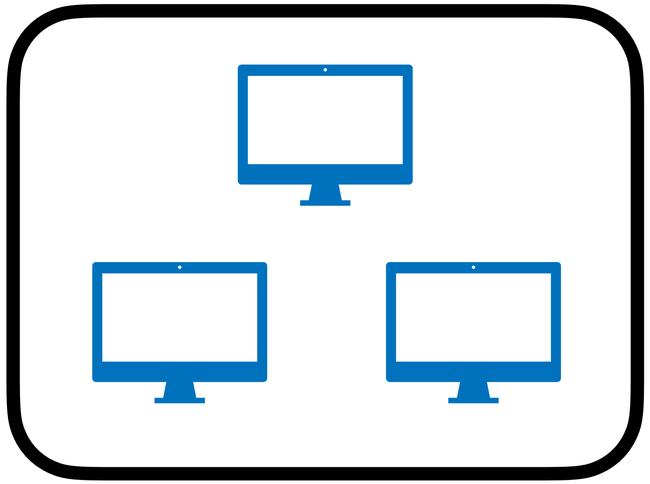
- Coverage of failure modes



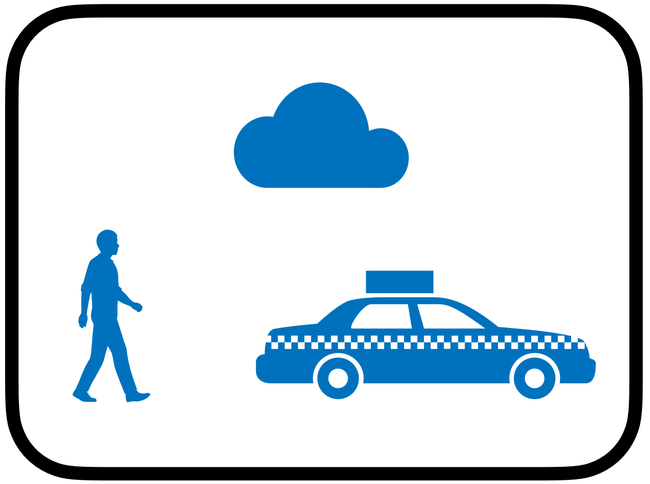
- Prioritization by likelihood



Components of the risk-based framework



Simulation



Generative models

Search algorithm

Search

$$p_\gamma := \mathbb{P}_0(f(X) < \gamma)$$

- Random search (aka naive Monte Carlo)

$$\hat{p}_\gamma = \frac{1}{N}\sum_{i=1}^{N} I\{f(x_i) < \gamma\}$$

Estimate

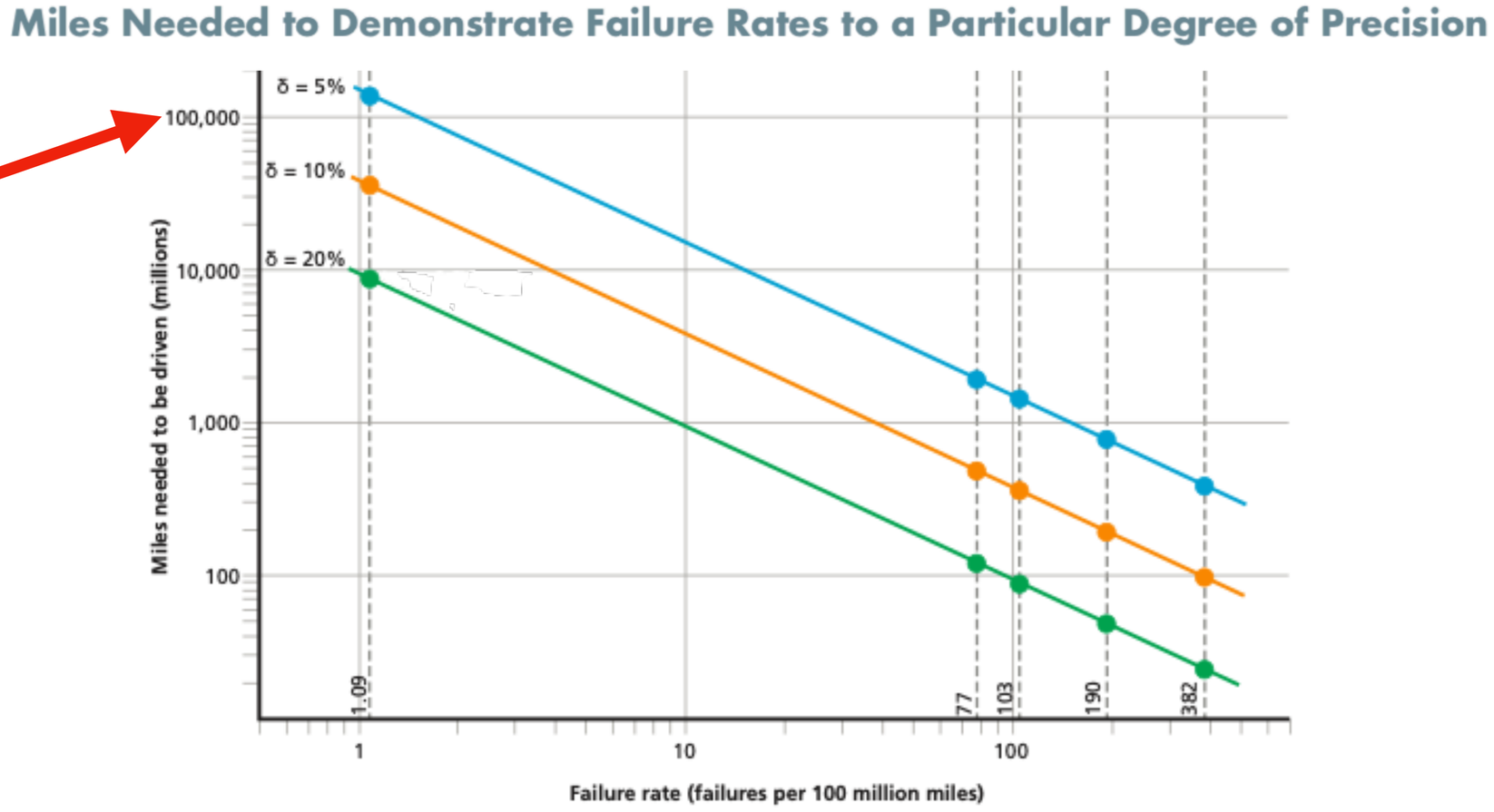
$$\mathbb{E}[(\hat{p}_\gamma / p_\gamma - 1)^2] = \frac{1 - p_\gamma}{N p_\gamma}$$

Error of estimate

- Rule of thumb: need at least $100/p_\gamma$ samples for accurate estimate (error bars < 10%)

Search

100 billion miles needed per code release!



[Kalra & Paddock 2016]

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Risk

Find failure modes and quantify the probability of failure

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- How do we solve it quickly?

Techniques for rare-event simulation

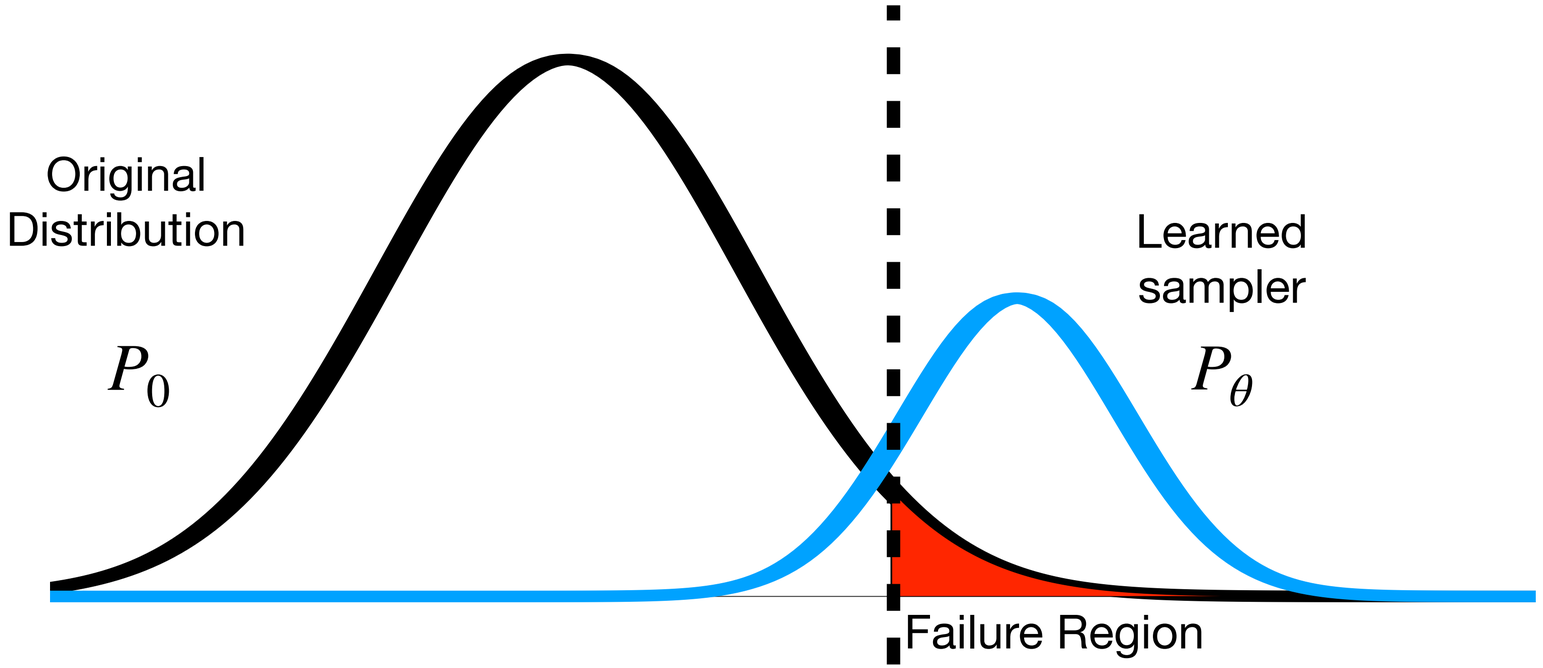
O’Kelly*, Sinha*, Namkoong*, Duchi, Tedrake. NeurIPS 2018.
Sinha*, O’Kelly*, Duchi, Tedrake. Under review.




Warm up: cross-entropy method

Goal: find a good parametric importance-sampling distribution

- Explore: draw samples from current sampling distribution
- Exploit: update sampling distribution



[Rubinstein & Kroese 2004, Asmussen & Glynn 2007]

Warm up: cross-entropy method

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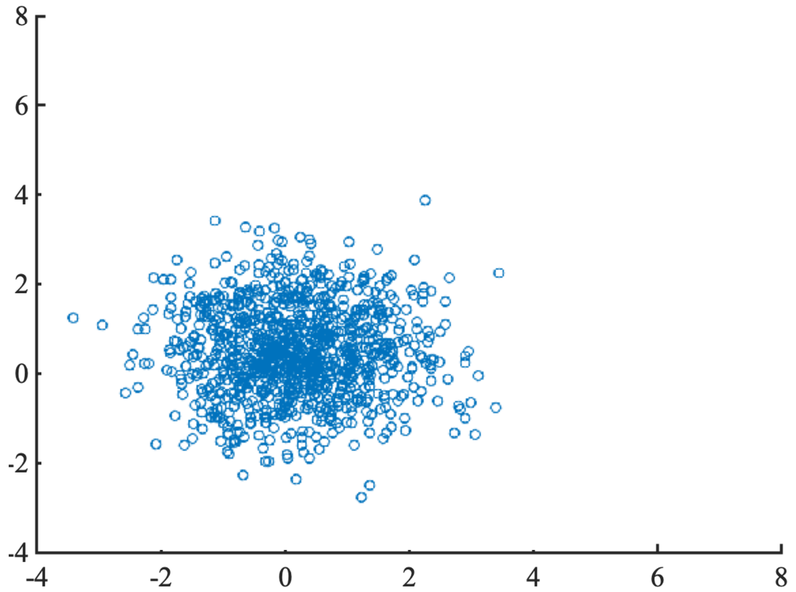
$$P_0 = \mathcal{N}(0, I)$$



Warm up: cross-entropy method

Goal: find a good parametric importance-sampling distribution

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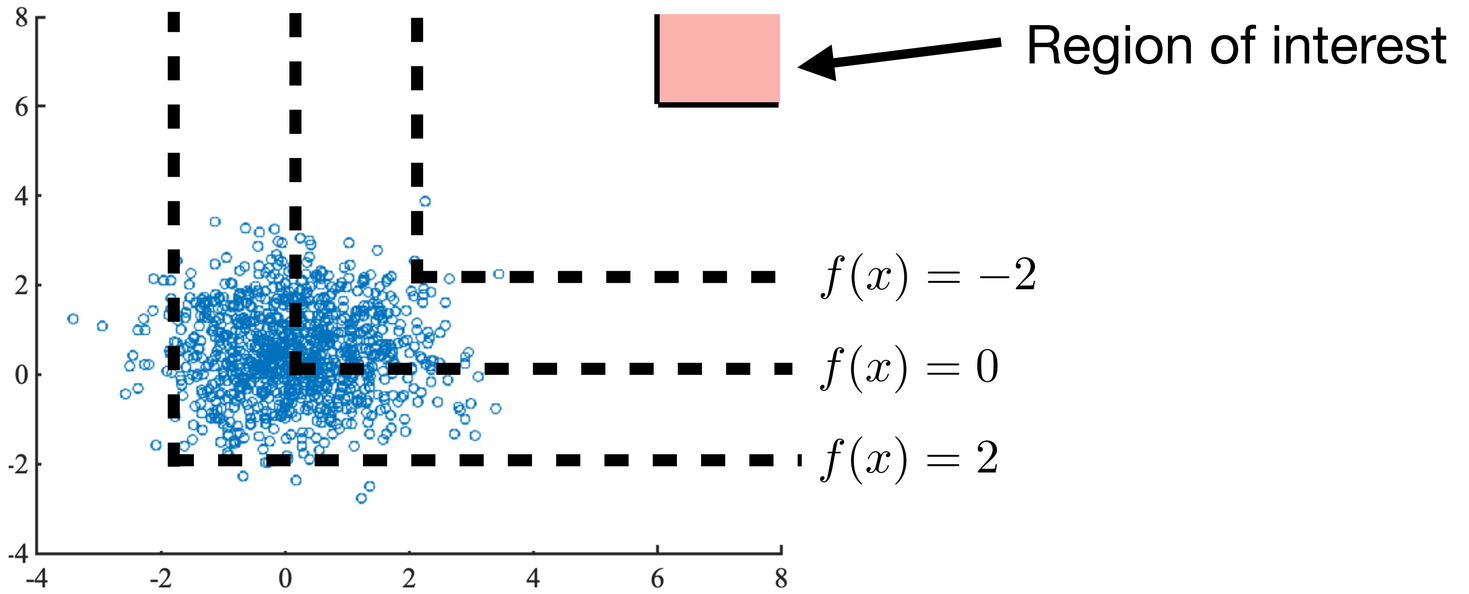
$$P_0 = \mathcal{N}(0, I)$$

$$f(x) = -\min(x_{[i]})$$

$$\gamma = -6$$



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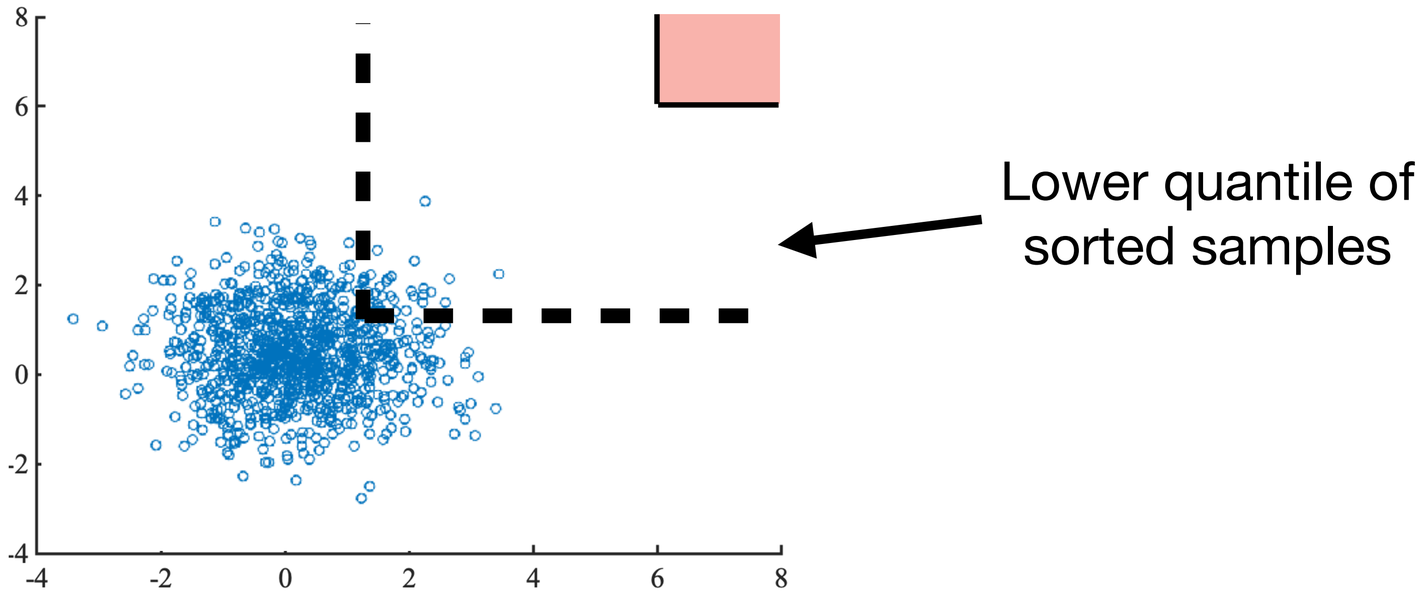

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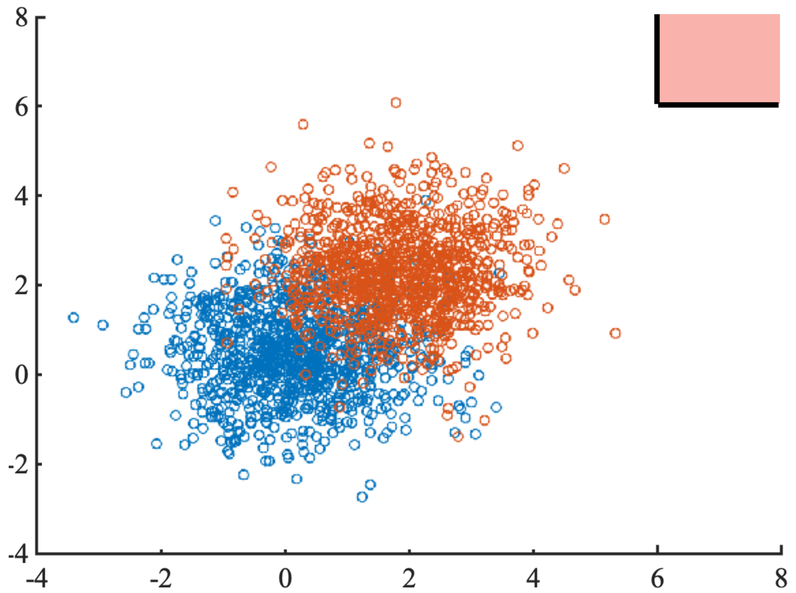
Sampling family: $\mathcal{N}(\theta, 1)$



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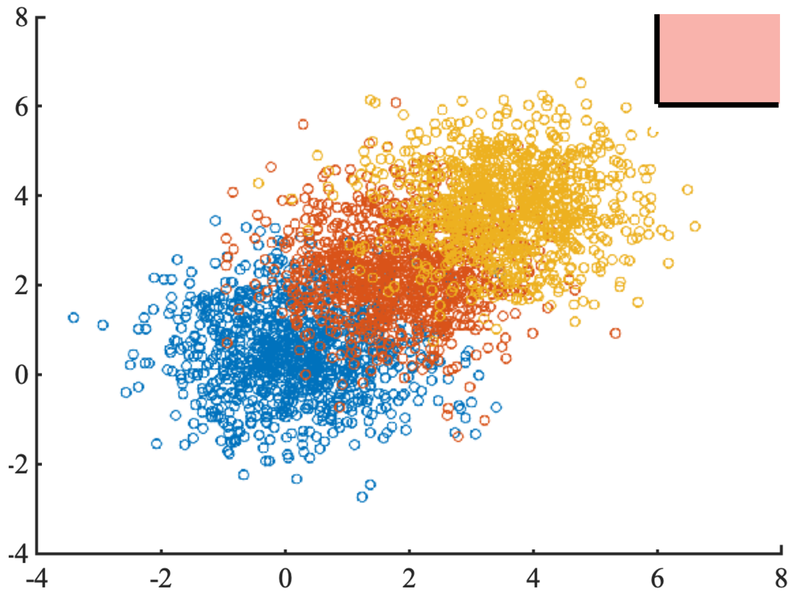

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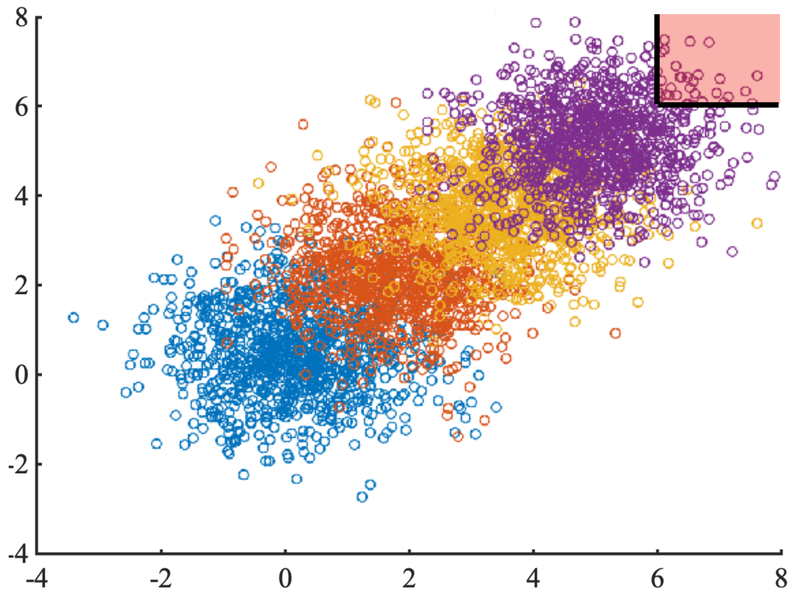

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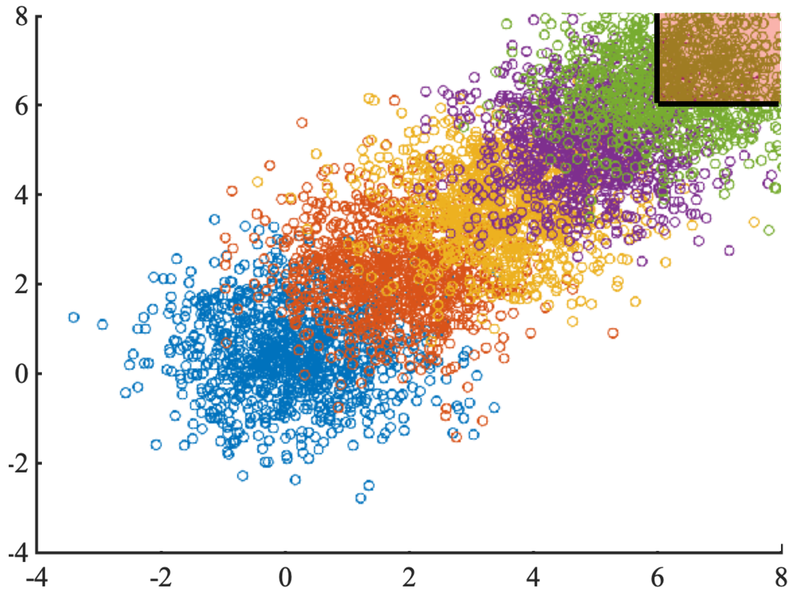
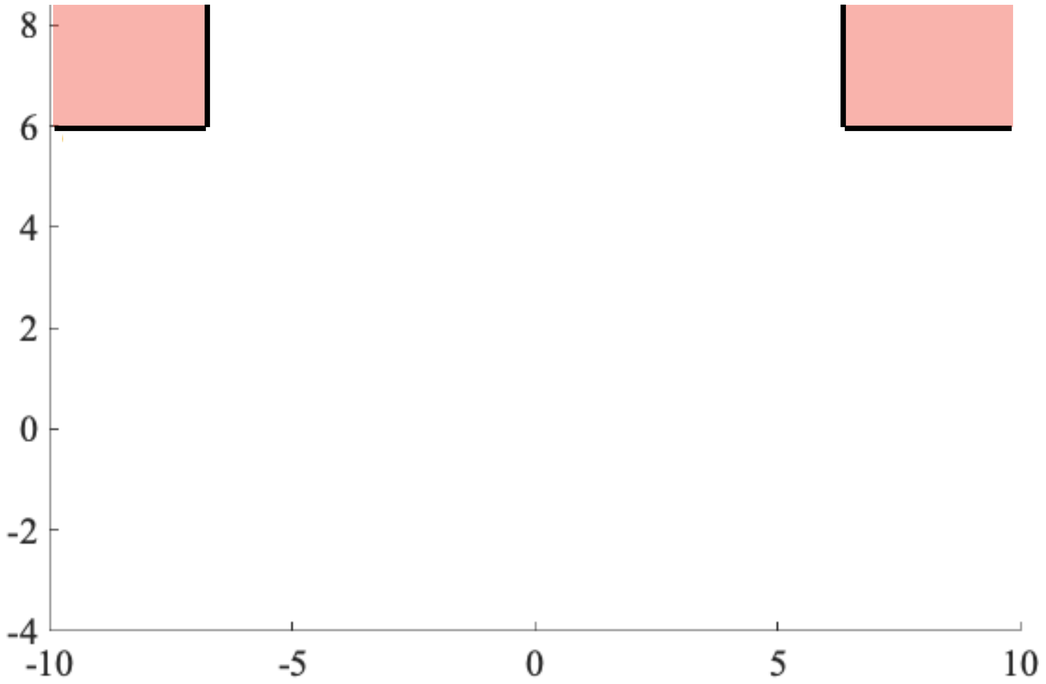


Warm up: cross-entropy method

Downside: need to know a good parametrization of samplers

$$f(x) = -\min(|x_{[1]}|, x_{[2]})$$



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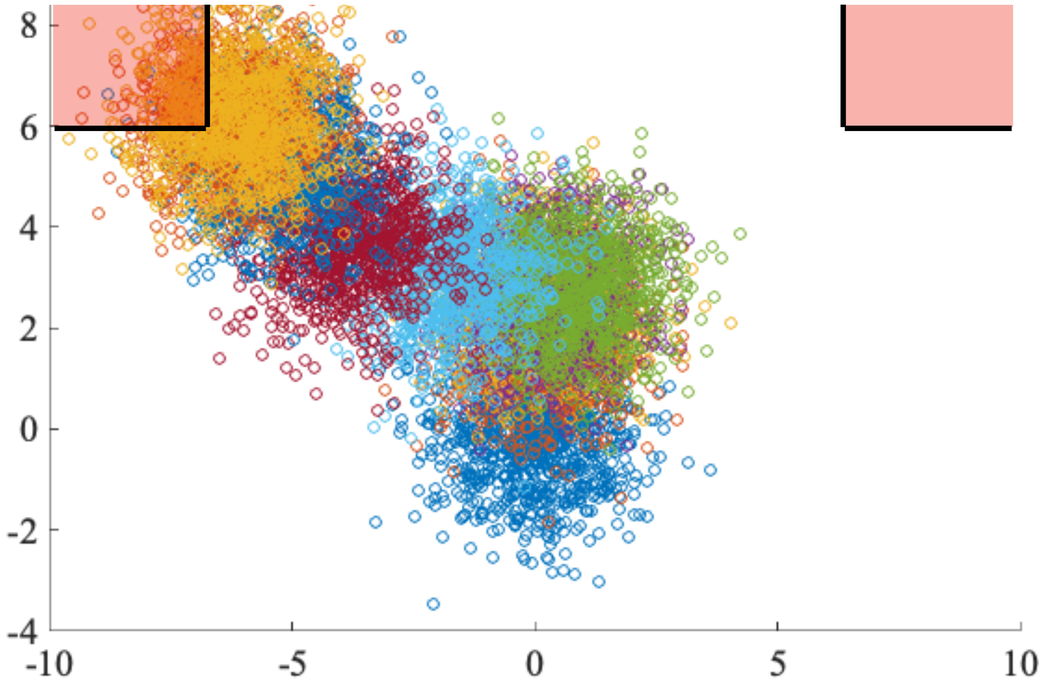

$\mathcal{N}(\theta, 1)$ family fails catastrophically!

Warm up 2: adaptive multilevel splitting (AMS)

Goal: decompose rare probability into a ladder of non-rare probabilities

- Exploit: throw away worst samples and rejuvenate from the remaining good ones
- Explore: MCMC to sample from updated level

$$\mathbb{P}_0(f(X) < \gamma) = \prod_{k=1}^{K} \mathbb{P}(f(X) < L_k | f(X) < L_{k-1})$$

$$\infty =: L_0 > L_1 \ldots > L_K := \gamma$$

[Guyader et al. 2011, Bréhier et al. 2015, Web et al. 2018, Norden et al. 2020]

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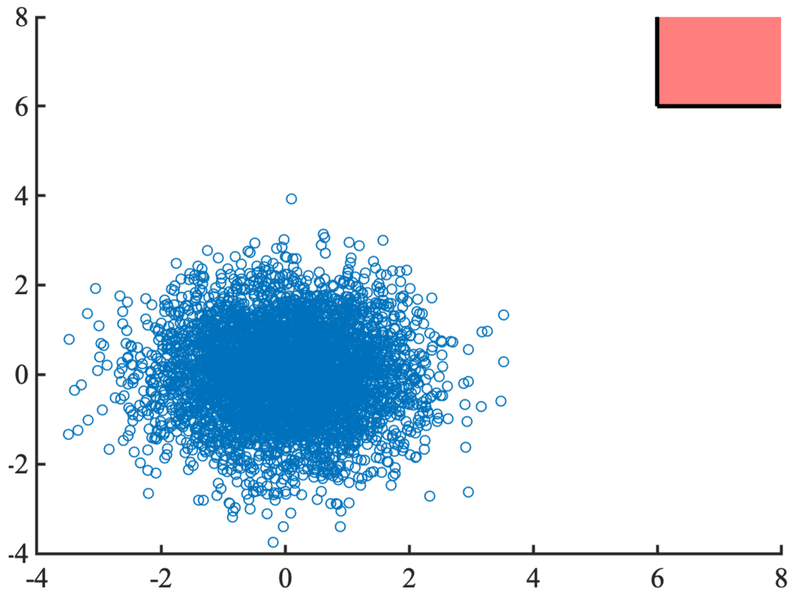

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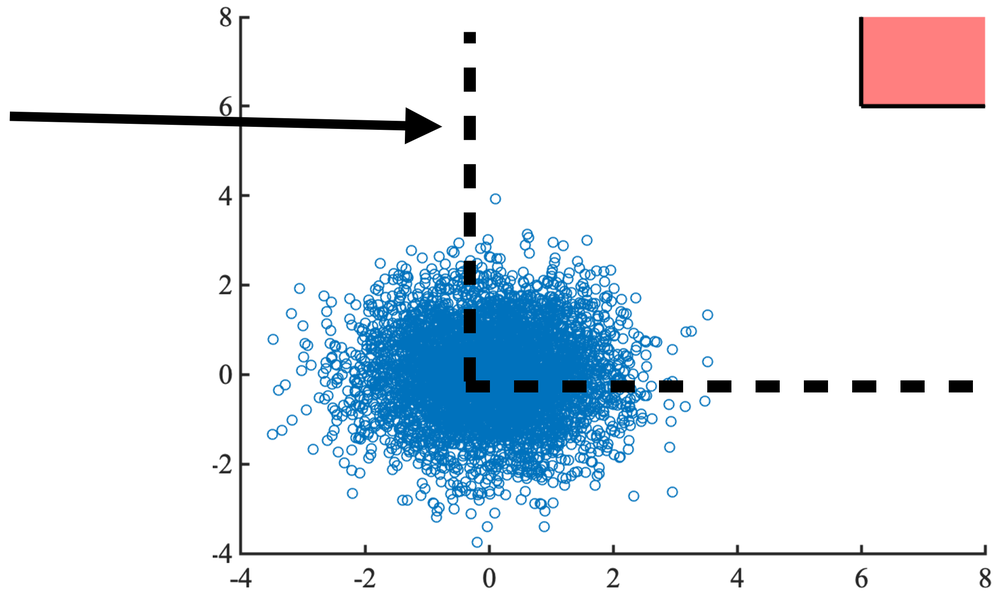
Empirical $(1-\delta)$-quantile

This is level L_1

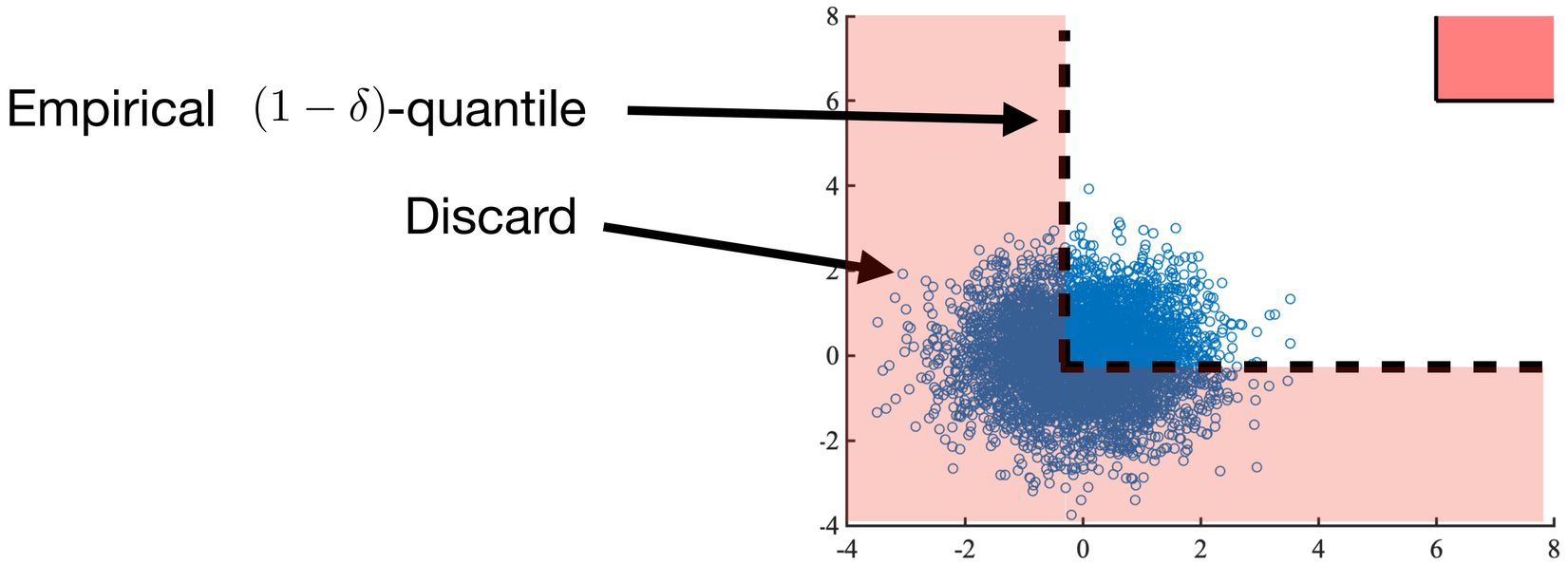


Warm up 2: adaptive multilevel splitting (AMS)

Goal: decompose rare probability into a ladder of non-rare probabilities

- Exploit: throw away worst samples and rejuvenate from the remaining good ones
- Explore: MCMC to sample from updated level

$$\mathbb{P}_0(f(X) < \gamma) = \prod_{k=1}^{K} \mathbb{P}(f(X) < L_k | f(X) < L_{k-1})$$



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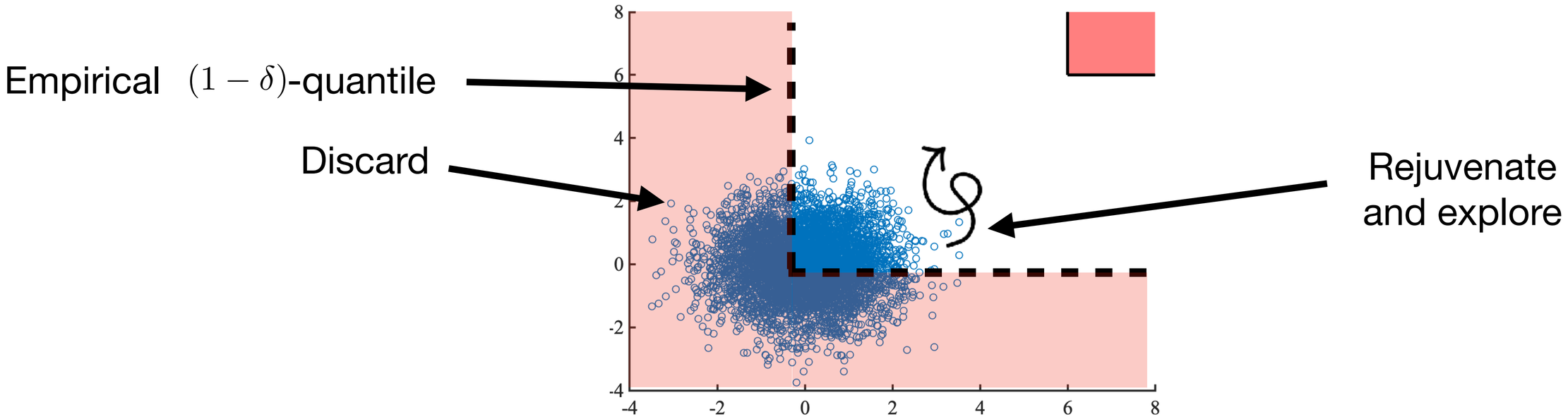

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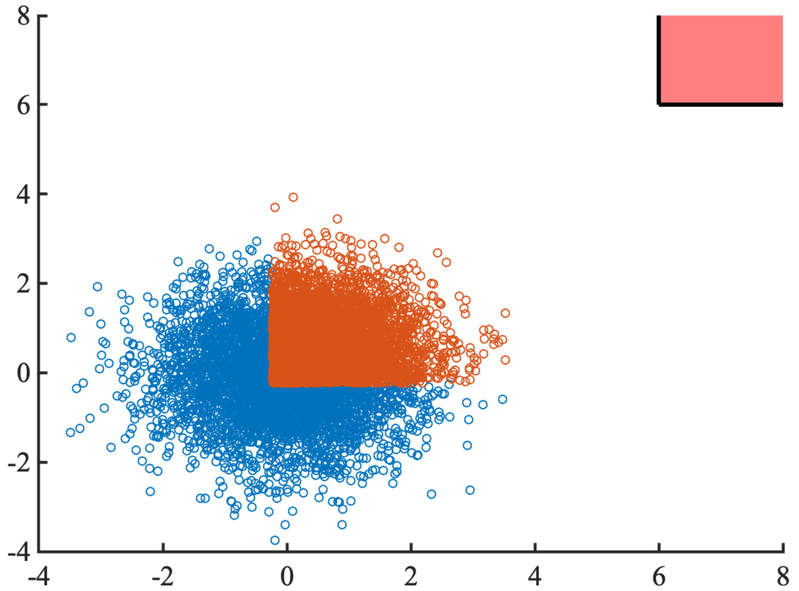

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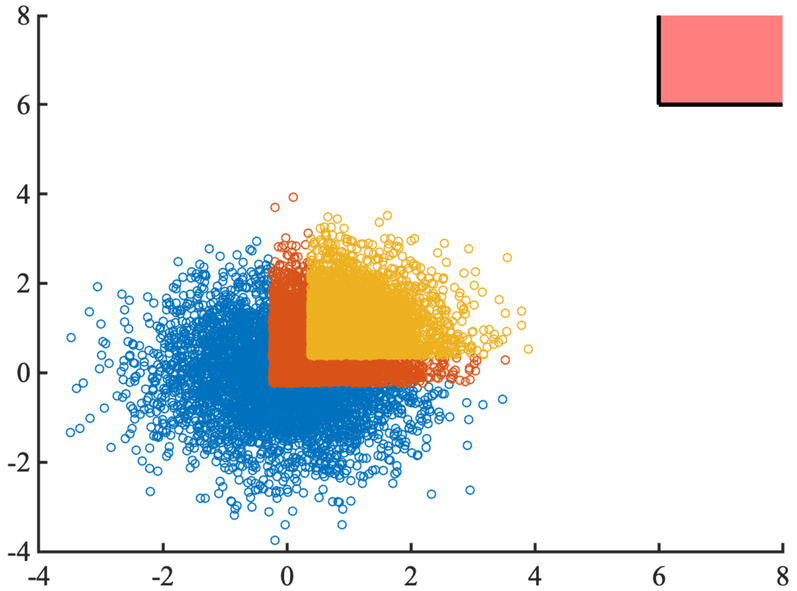

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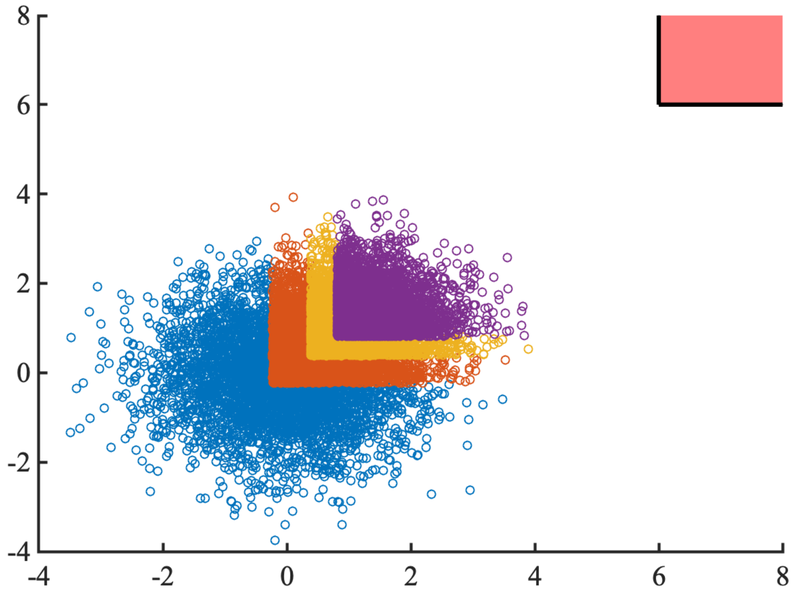

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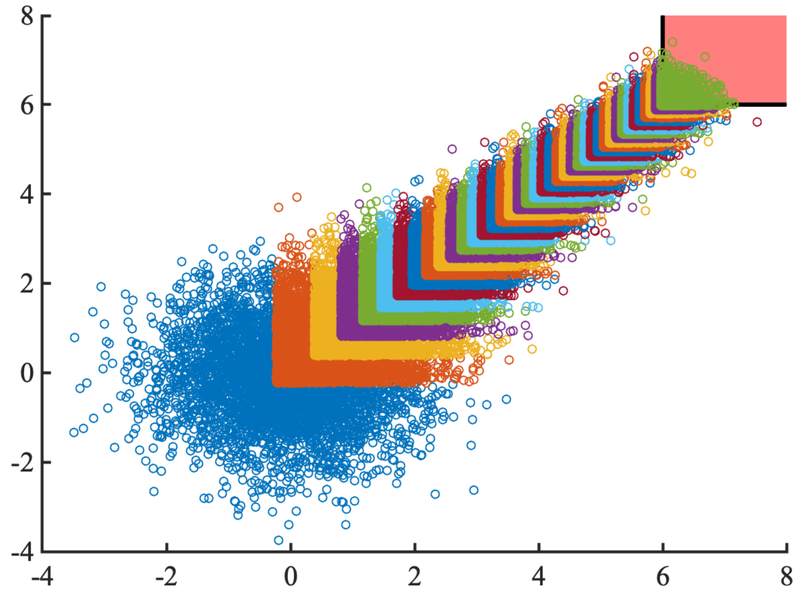
Random stopping time K

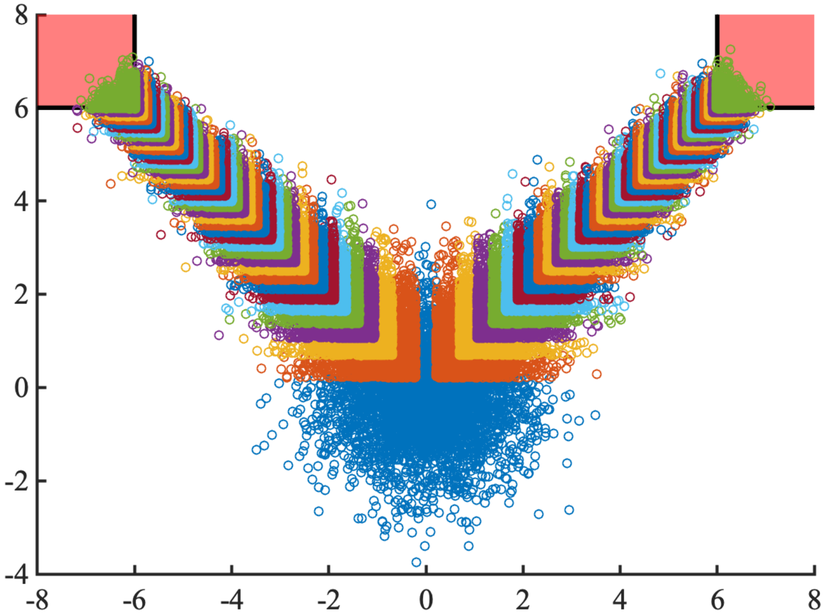
$$\hat{p}_\gamma = (1-\delta)^K$$



Warm up 2: adaptive multilevel splitting (AMS)

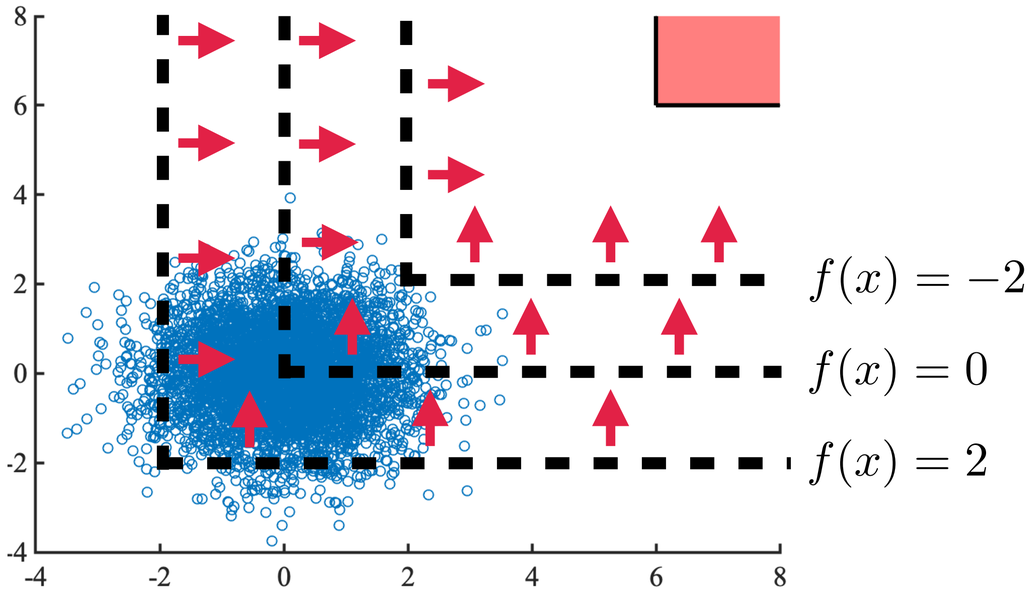
Upside

No need to come up with parametric distributions



Downside

Doesn’t use gradient information $\nabla f(x)$



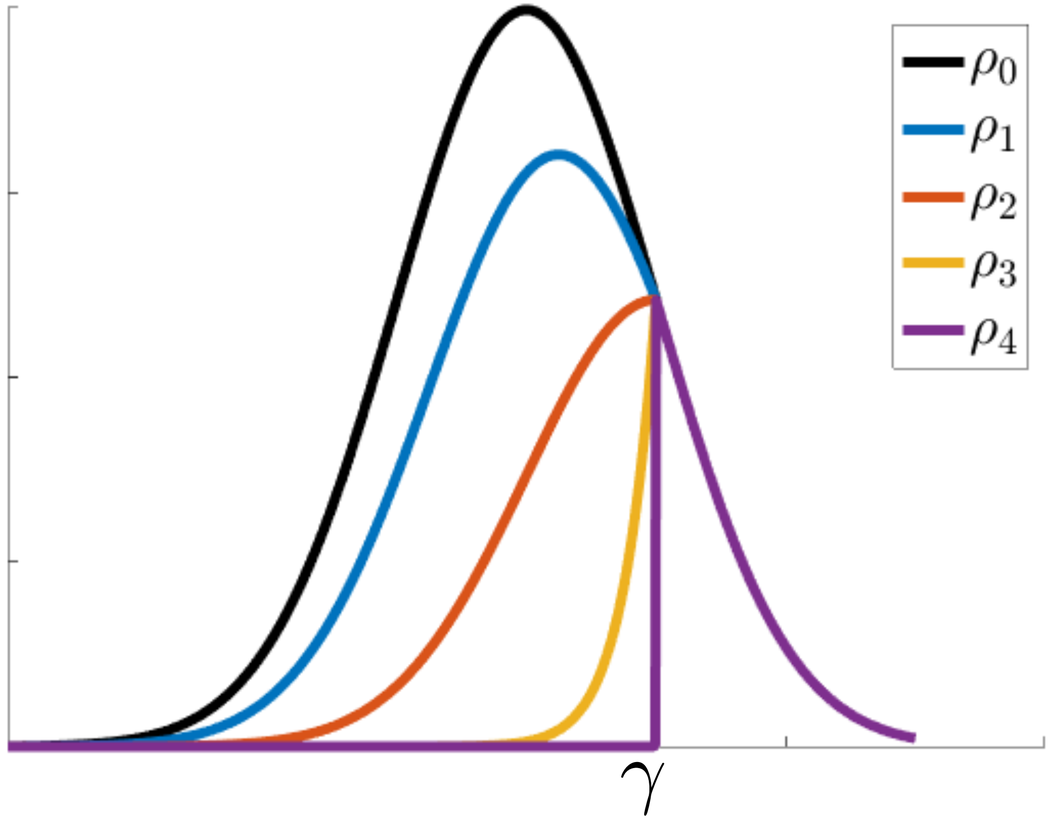
Challenge: intermediate distributions don’t depend smoothly on f

Our approach

- A smoother ladder towards failure

$$\rho_k(x) := \rho_0(x) \underbrace{\exp\left(-\beta_k \left[f(x) - \gamma\right]_+\right)}_{\text{exponential barrier}}$$

$$Z_k := \int_{\mathcal{X}} \rho_k(x) dx$$



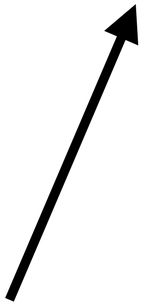
- Contrast with AMS $\rho_k(x) := \rho_0(x) \underbrace{I\{f(x) < L_k\}}_{\text{hard barrier}}$

Our approach

- A smoother ladder towards failure

$$\rho_k(x) := \rho_0(x) \underbrace{\exp\left(-\beta_k \left[f(x) - \gamma\right]_+\right)}_{\text{exponential barrier}}, \quad Z_k := \int_{\mathcal{X}} \rho_k(x) dx$$

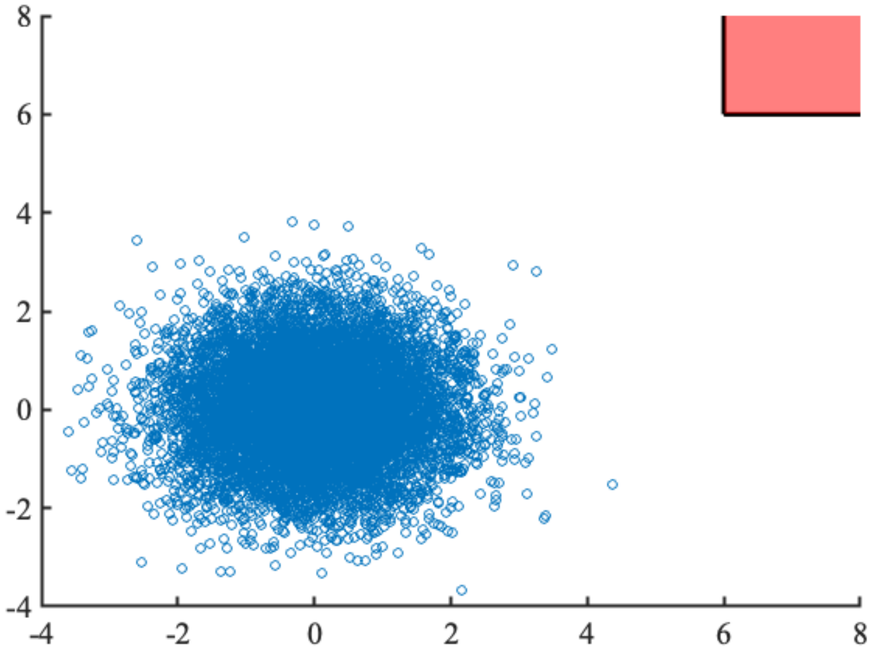
$$\mathbb{P}_0(f(X) < \gamma) = \mathbb{E}_{P_K}\left[\frac{Z_K}{Z_0}\frac{\rho_0(X)}{\rho_K(X)} I\{f(X) < \gamma\}\right], \qquad \frac{Z_K}{Z_0} = \prod_{k=1}^{K} \frac{Z_k}{Z_{k-1}}$$



We will estimate these

Our approach

- A smoother ladder towards failure
- Exploit: determine the next β using current samples (k^{th} distribution)
- Explore + optimize: utilize gradient-based MCMC to sample from $(k+1)^{st}$ distribution
- Estimate: compute Z_{k+1}/Z_k via bridge sampling



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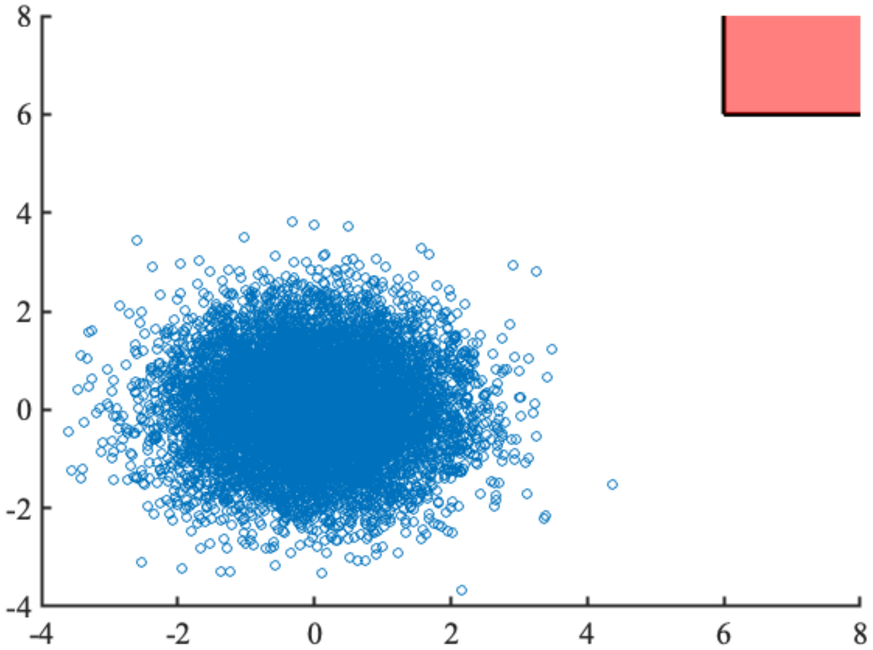
Choose β_{k+1} such that

$$\frac{Z_{k+1}}{Z_k} \approx \alpha$$

(quasiconvex problem)



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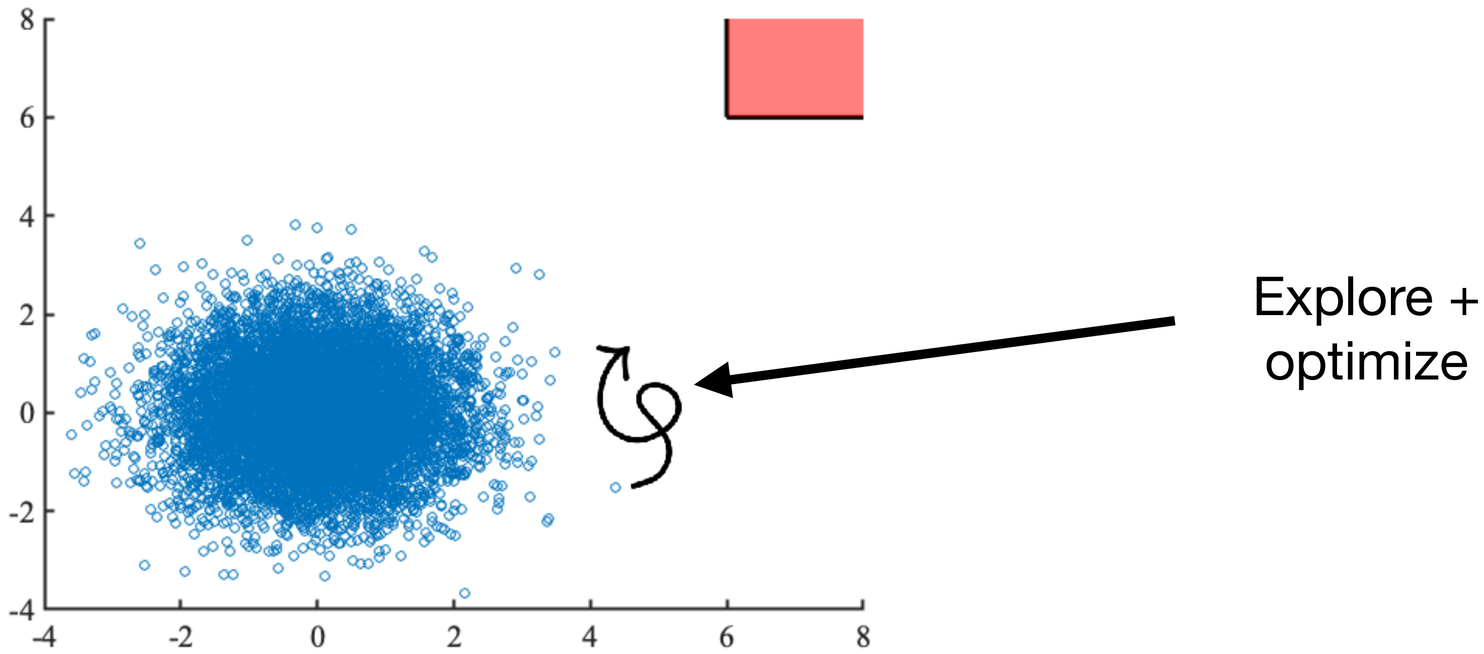
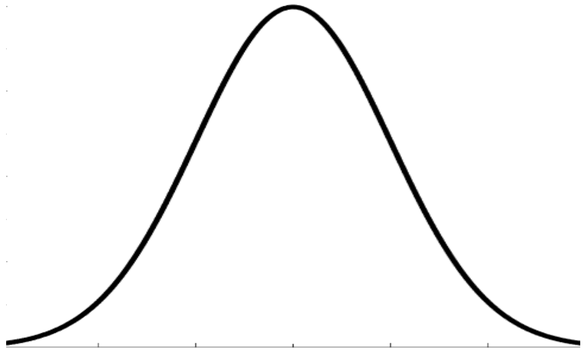
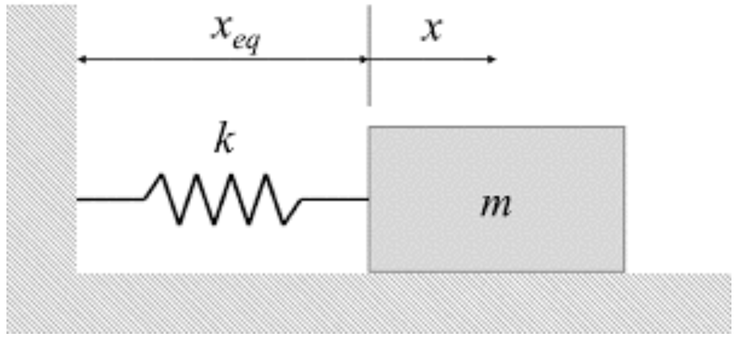


Hamiltonian Monte Carlo (HMC)

- Treat $-\log \rho_k$ as a physical energy potential and simulate the dynamics (ODEs)



Gaussian distribution



Spring-mass system

- Automatic tradeoff between exploration and optimization

$$\nabla \log \rho_k(x) = \underbrace{\nabla \log \rho_0(x)}_{\text{exploration}} - \beta_k \underbrace{\nabla f(x) I\{f(x) > \gamma\}}_{\text{optimization}}$$

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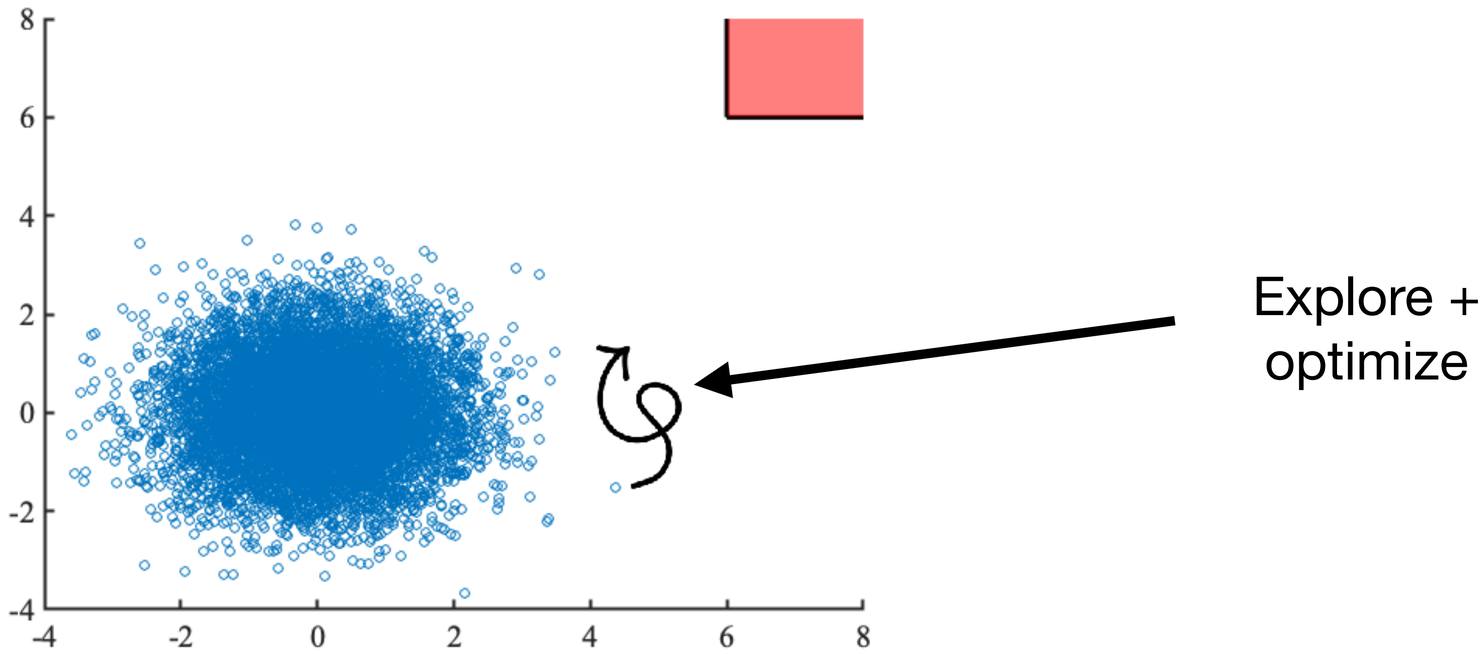

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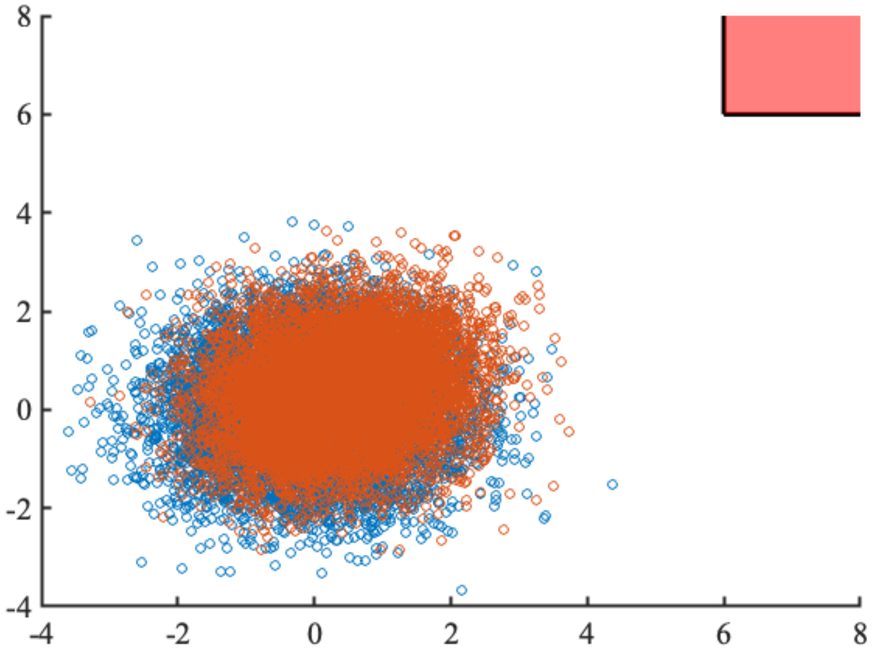

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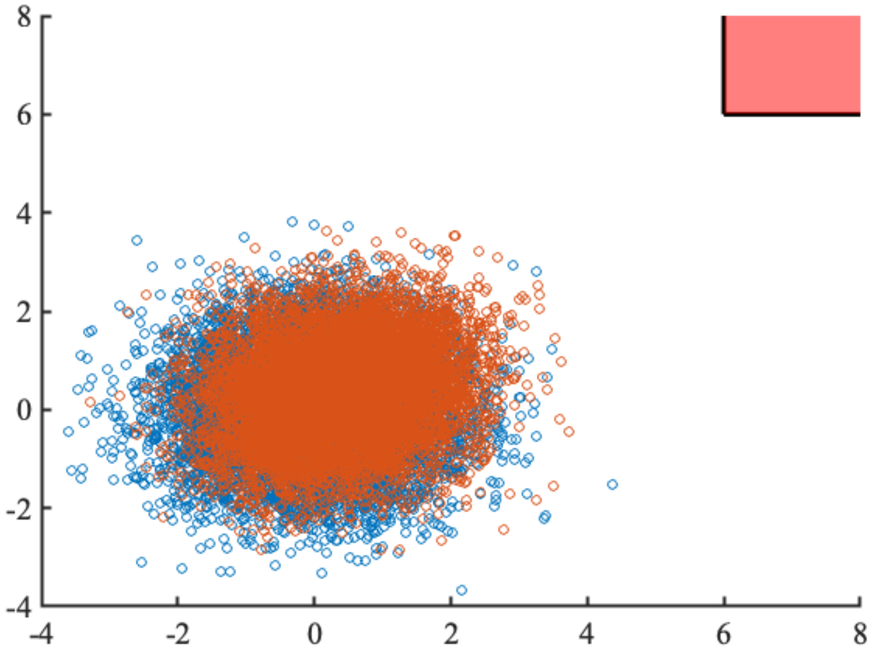
Use both sets of samples to compute an accurate estimate of Z_{k+1}/Z_k

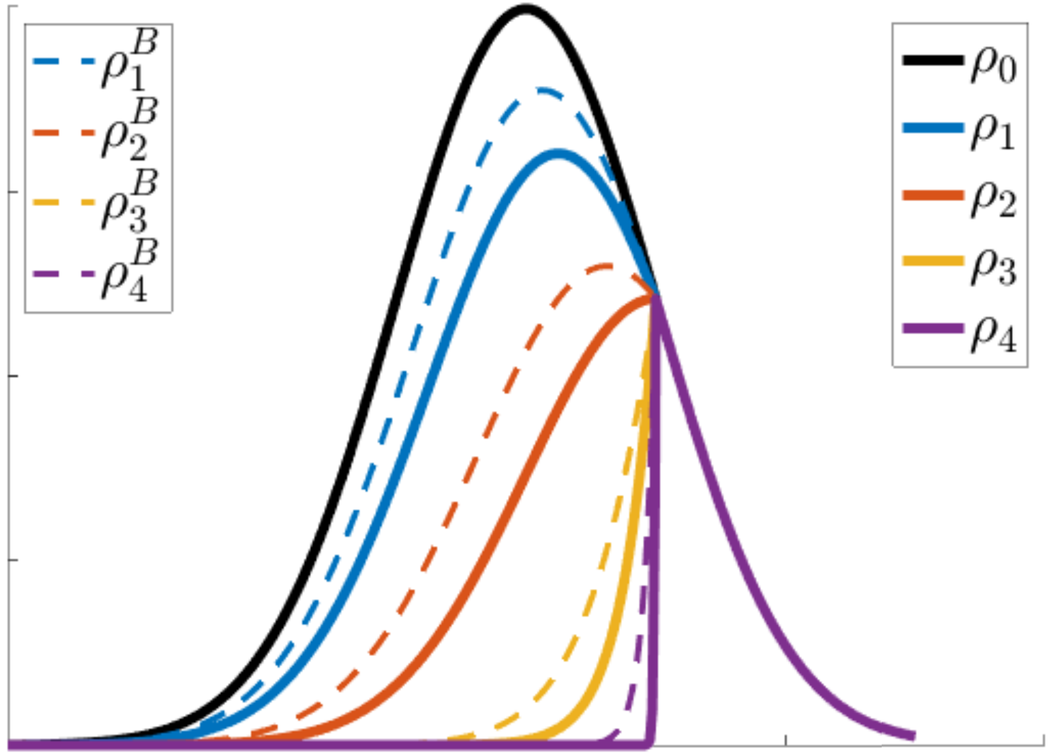


Bridge sampling

- Use samples from neighboring distributions to estimate their ratio Z_{k+1}/Z_k

$$\frac{Z_{k+1}}{Z_k} = \frac{Z_k^B/Z_k}{Z_k^B/Z_{k+1}}$$

Use an auxiliary “bridge” distribution



[Bennett 1976, Meng & Wong 1996]

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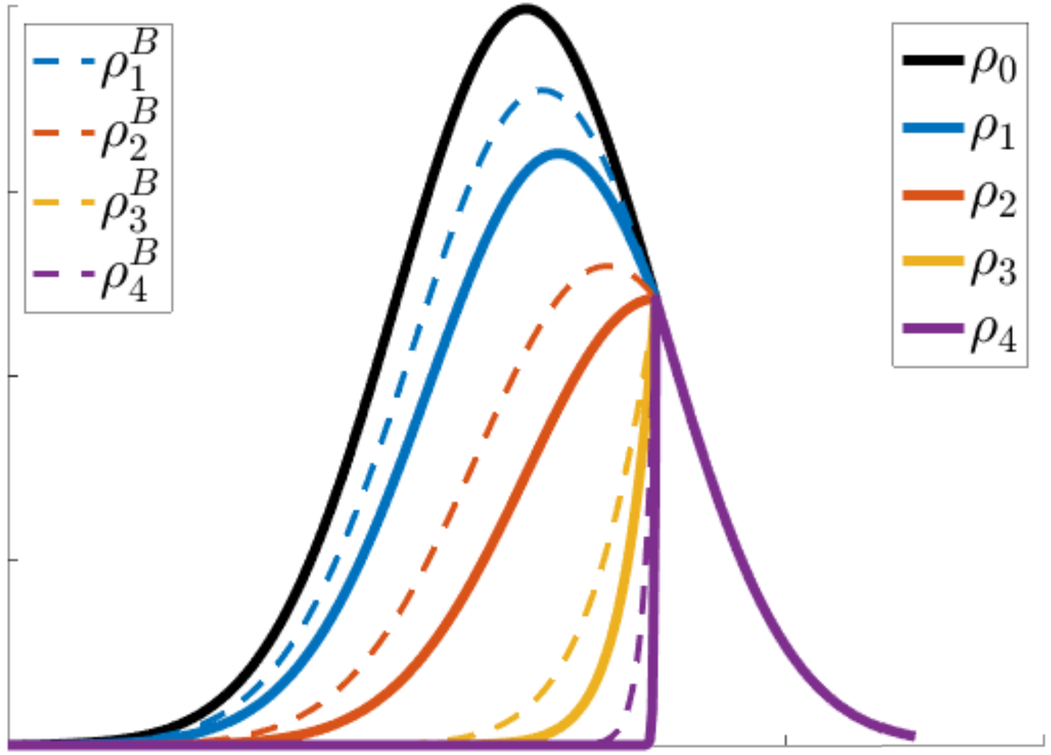
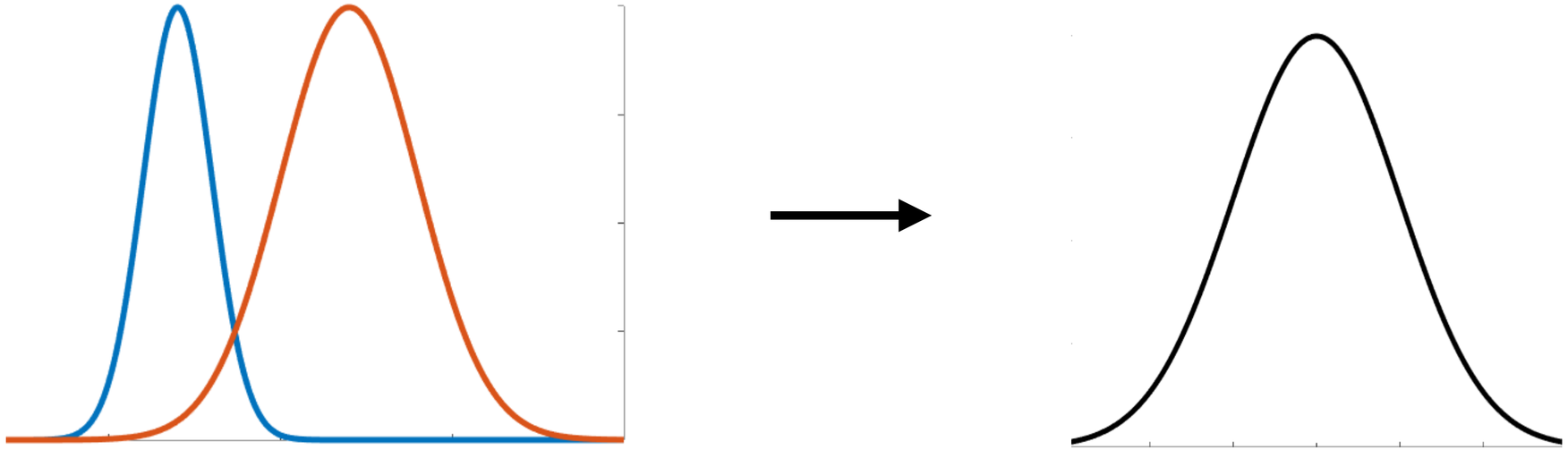


Problem:
Error depends on distance between distributions

[Bennett 1976, Meng & Wong 1996]

Neural warping

- Error of bridge-sampling estimate depends on the distance between distributions
- Transform the space so they are closer (“warp” the space between them)



- Classical techniques: mean shift, affine scaling [Voter 1985, Meng & Schilling 2002]
- Modern ML toolbox: normalizing flows [Papamakarios et al. 2019]
- Bonus: warping helps HMC [Girolami & Calderhead 2011, Hoffman et al. 2019]

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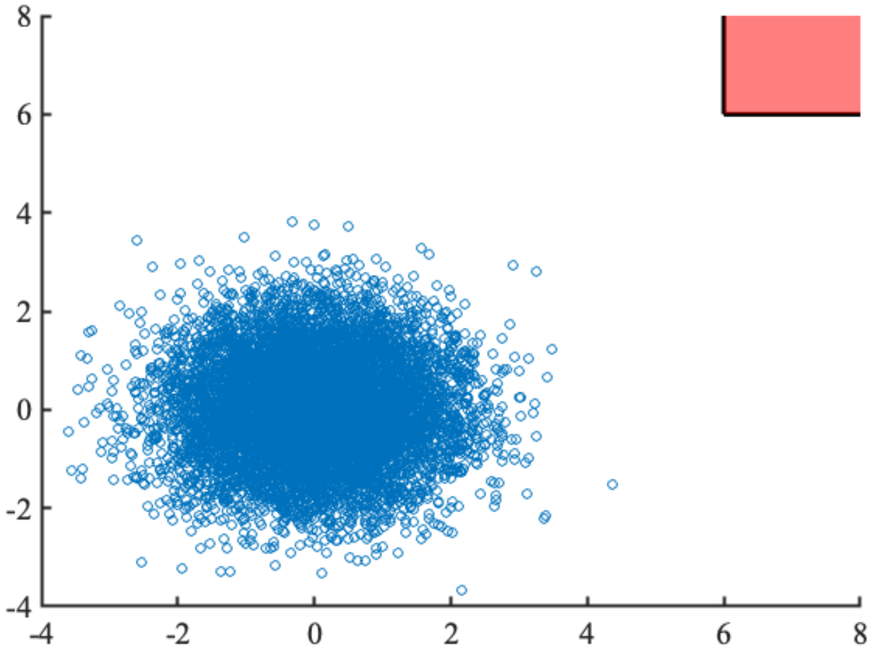

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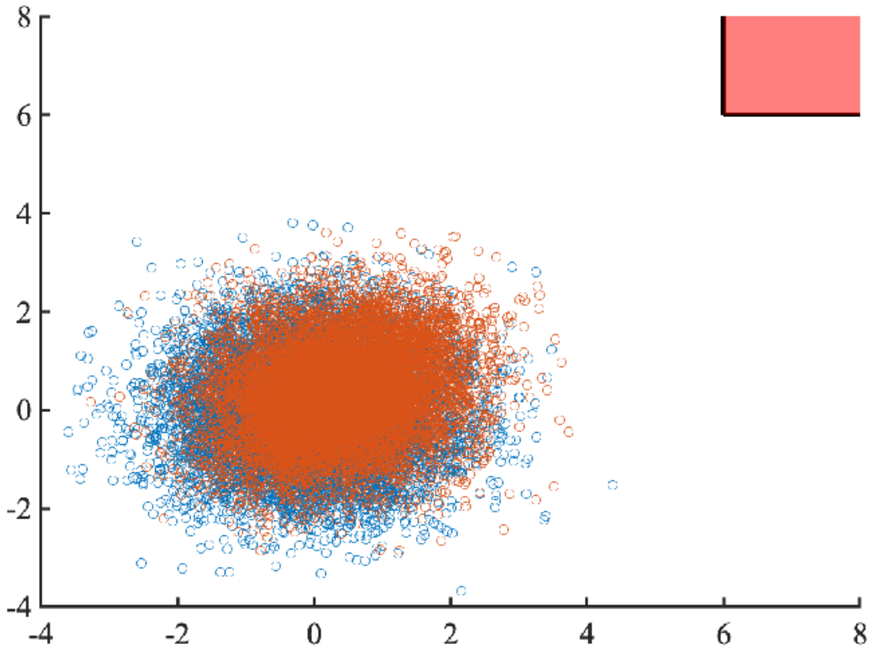

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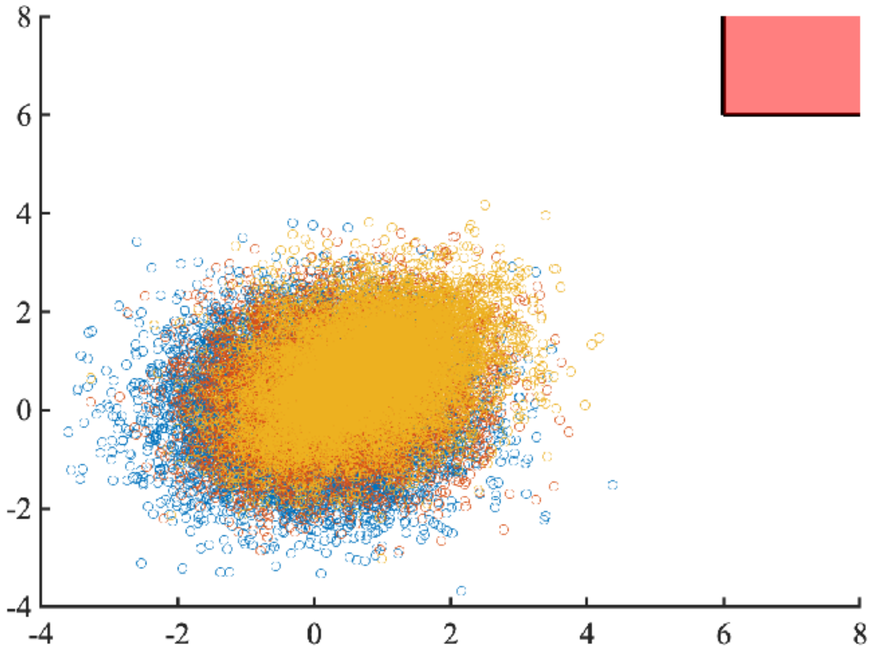

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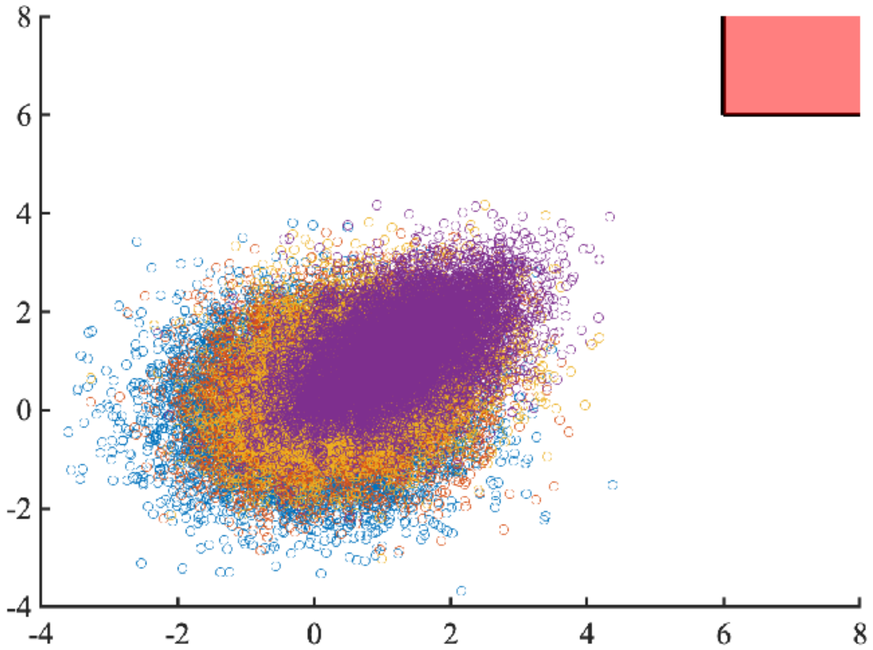

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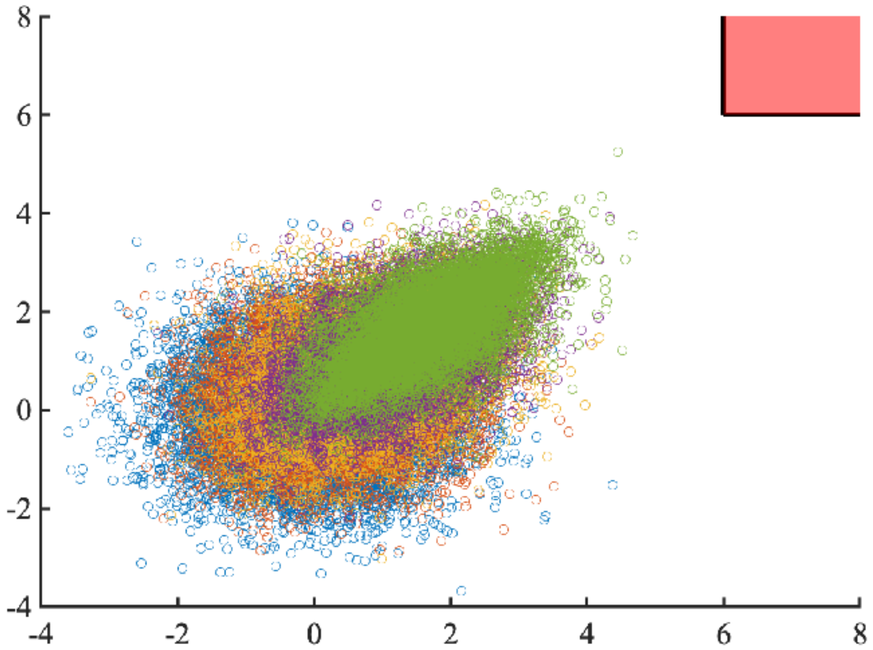

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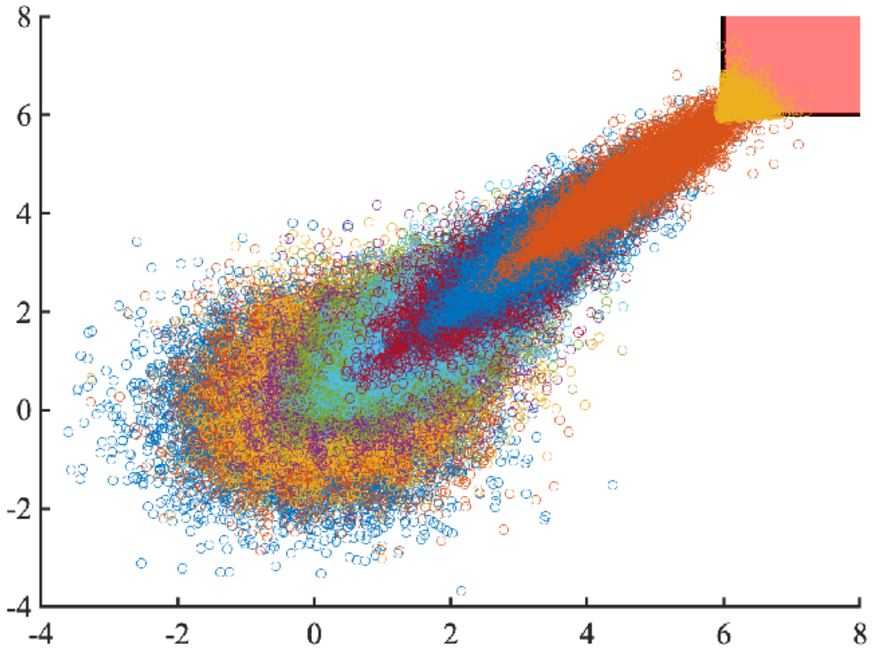

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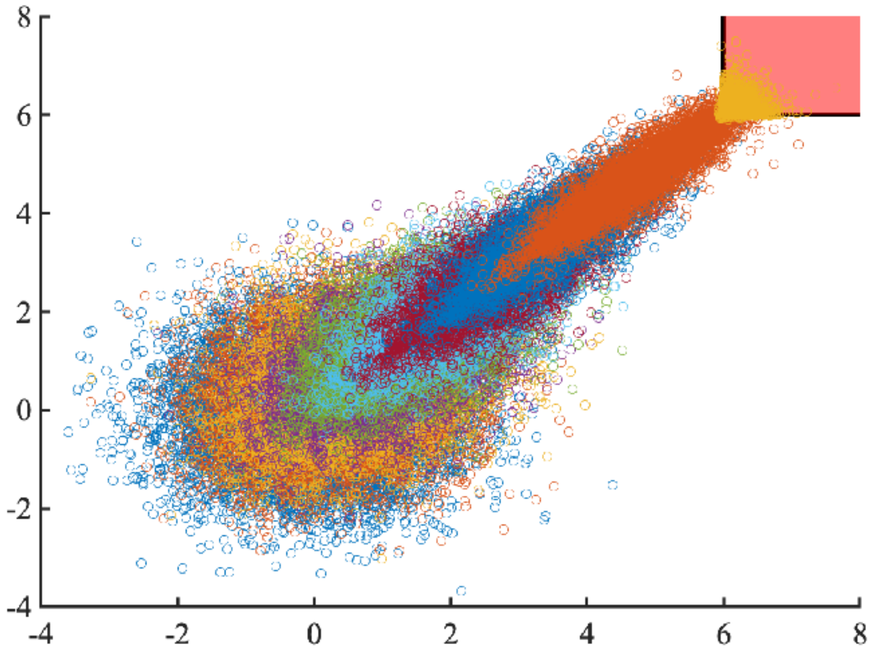
Random stopping time K

$\hat{p}_\gamma \approx \alpha^K$



Performance guarantees

- Theorem: Number of iterations $K \to \log p_\gamma / \log \alpha$ and $\mathbb{E}[(\hat{p}_\gamma / p_\gamma - 1)^2] \leq 2KD/N$
  - D depends on (warped) distance between consecutive distributions

- Computational cost is $O(KN)$ simulations

	Cost	Error
Neural bridge sampling	$N \log(1/p_\gamma)$	$\dfrac{\log(1/p_\gamma)}{N}$
Monte Carlo	N	$\dfrac{1}{p_\gamma N}$

Performance guarantees

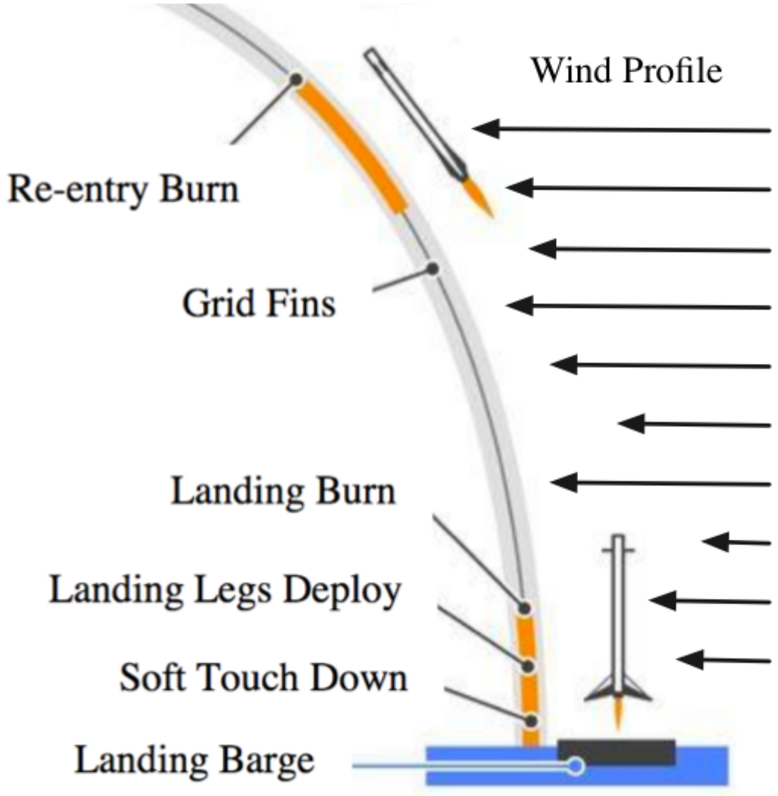
- Overall efficiency gain of $O\left(\frac{1}{p_\gamma \log(p_\gamma)^2}\right)$ over Monte Carlo

- Relative advantage scales with rarity

	Time	Error
Neural bridge sampling	$N \log(1/p_\gamma)$	$\frac{\log(1/p_\gamma)}{N}$
Monte Carlo	N	$\frac{1}{p_\gamma N}$

Experiments: rocket design

- Vertical landing of an orbital-class rocket (e.g. SpaceX Falcon 9)
- P_0 is the model of wind gusts during flight (100 dimensions)
- $f(x)$ is the distance from the center of the launchpad at landing



Rocket1: Boosters capable of 15% main thrust

Rocket2: Boosters capable of 10% main thrust

Bigger boosters are safer but mean smaller payloads

Experiments: rocket design

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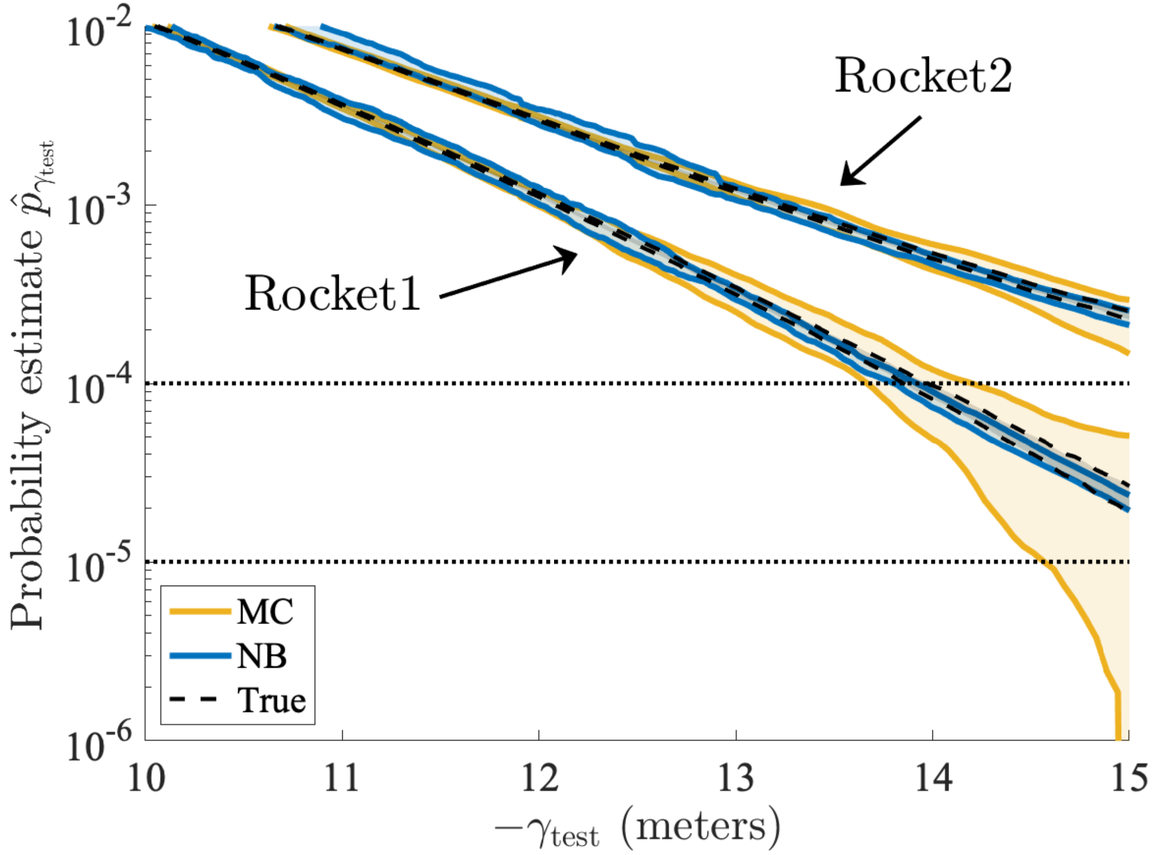
MC/NB use 100,000 samples

True is MC with 50M samples



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- Vertical landing of an orbital-class rocket (e.g. what SpaceX does)
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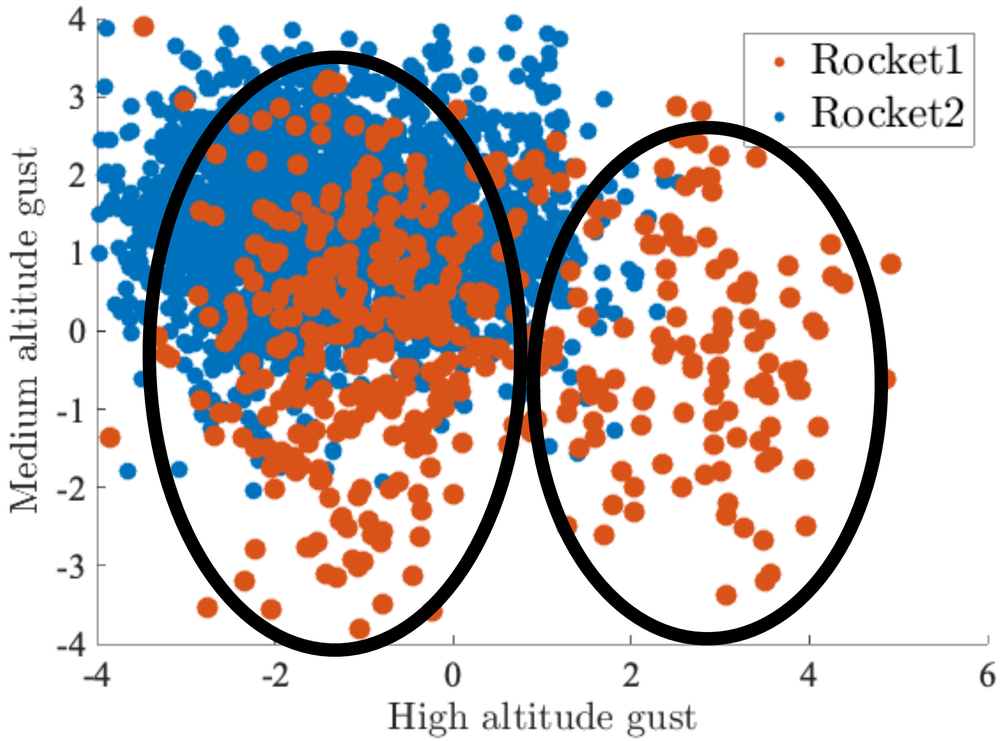
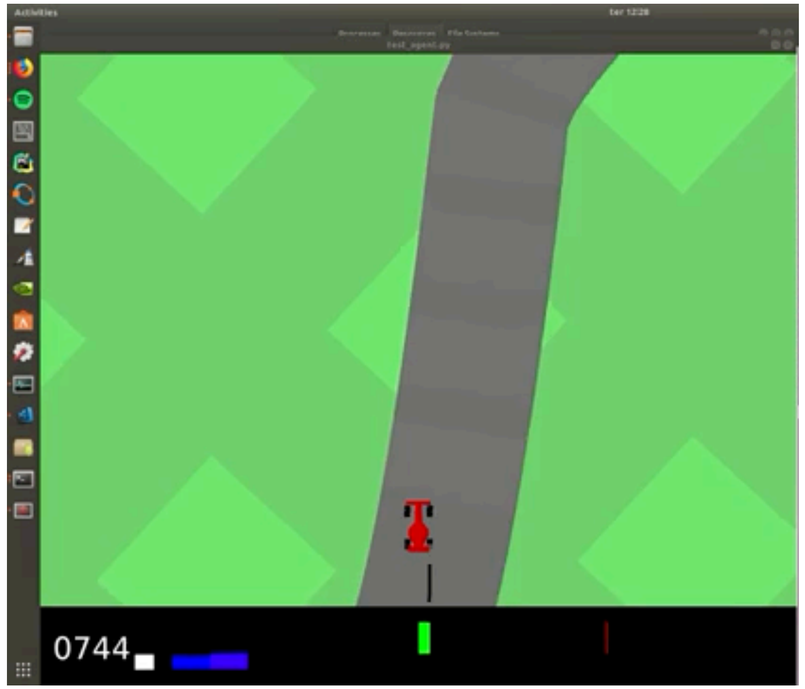
PCA visualization of failure modes



Experiments: OpenAI CarRacing

- Challenging environment (pixels to actions)
- P_0 is the model for track generation (24 dimensions)
- $f(x)$ is the score achieved



Compare 2 SOTA policies:

	Average Score (over 2M runs)
AttentionAgent [Tang et al. 2020]	903 ± 49
WorldModel [Ha & Schmidhuber 2018]	899 ± 46

Experiments: OpenAI CarRacing

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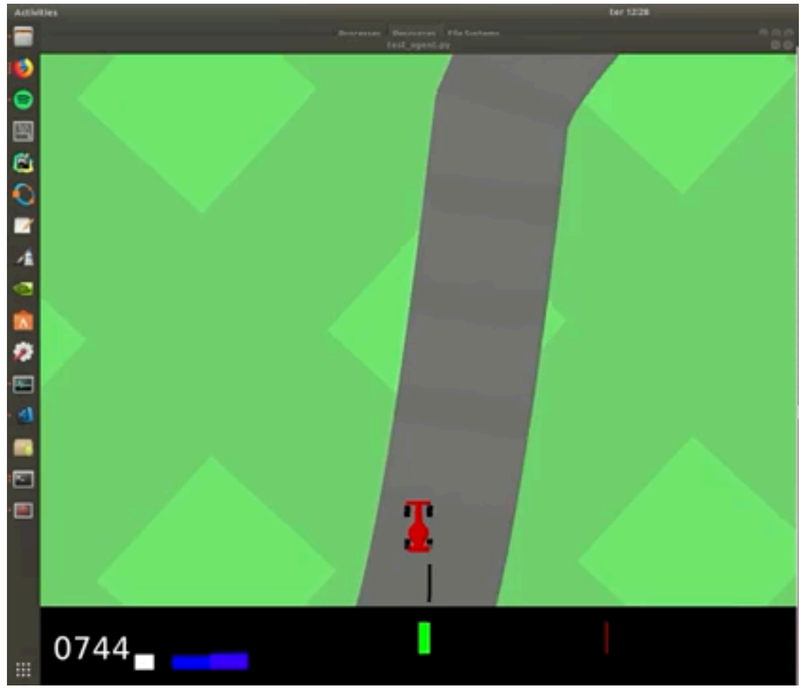
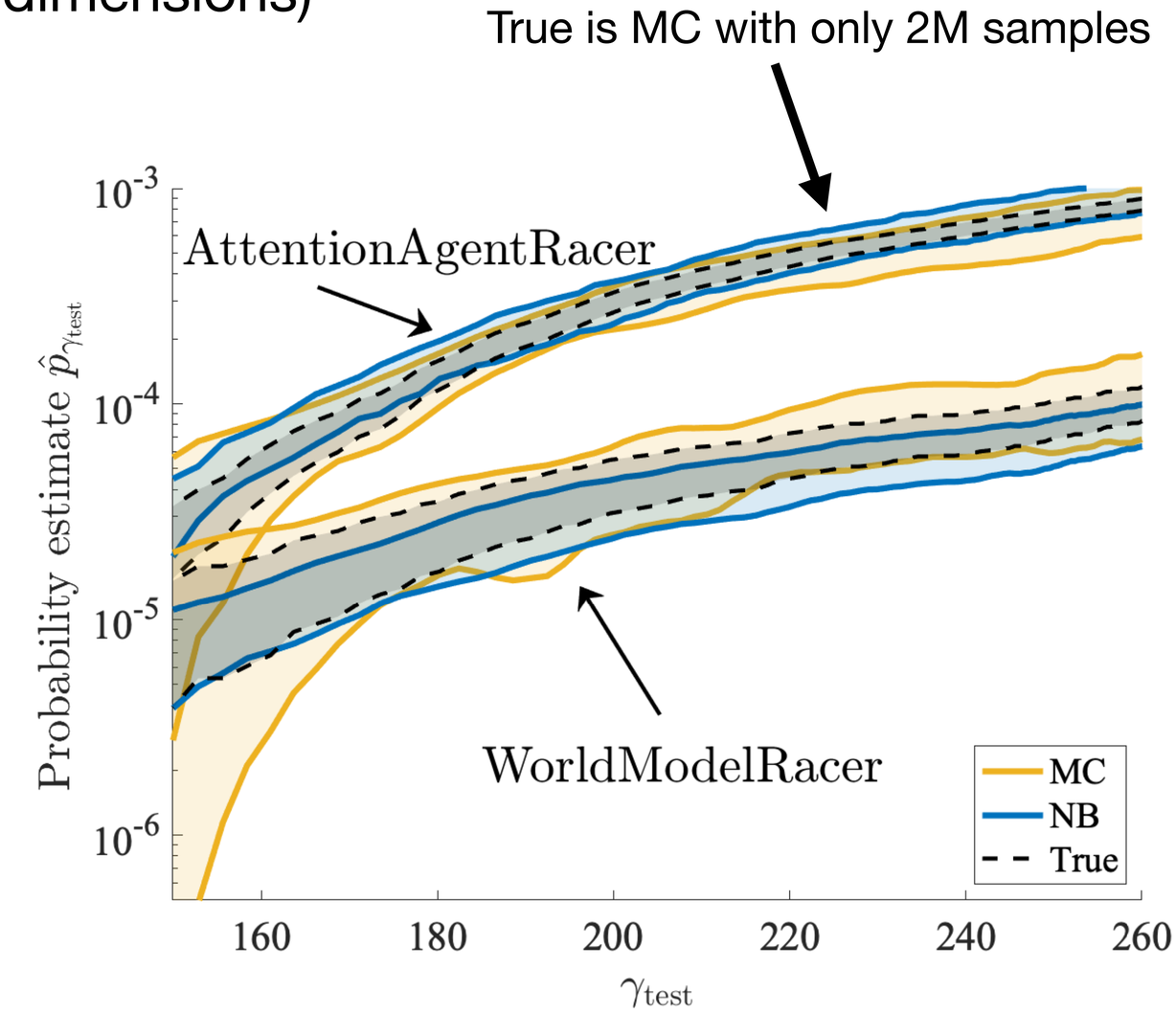




Experiments

Relative mean-square error $\mathbb{E}[(\hat{p}_\gamma/p_\gamma - 1)^2]$ over 10 trials

	Synthetic	MountainCar	Rocket1	Rocket2	AttentionAgentRacer	WorldModelRacer
MC	1.1821	0.2410	1.1039	0.0865	1.0866	0.9508
AMS	0.0162	0.5424	0.0325	0.0151	1.0211	0.8177
B	0.0514	0.3856	0.0129	0.0323	0.9030	0.7837
NB	**0.0051**	**0.0945**	**0.0102**	**0.0078**	**0.2285**	**0.1218**
p_γ	$3.6 \cdot 10^{-6}$	$1.6 \cdot 10^{-5}$	$2.3 \cdot 10^{-5}$	$2.4 \cdot 10^{-4}$	$\approx 2.5 \cdot 10^{-5}$	$\approx 9.5 \cdot 10^{-6}$

Neural bridge sampling outperforms other methods

Key ideas of this talk

Robustness

Build models with guaranteed performance over uncertainty sets

$$\min_{\theta \in \Theta} \max_{Q \in \mathcal{P}} \mathbb{E}_Q[f(\theta; X)]$$

- With small $\mathcal{P}$, can we solve this quickly?
- What about when $\mathcal{P}$ is large/unknown?

Risk

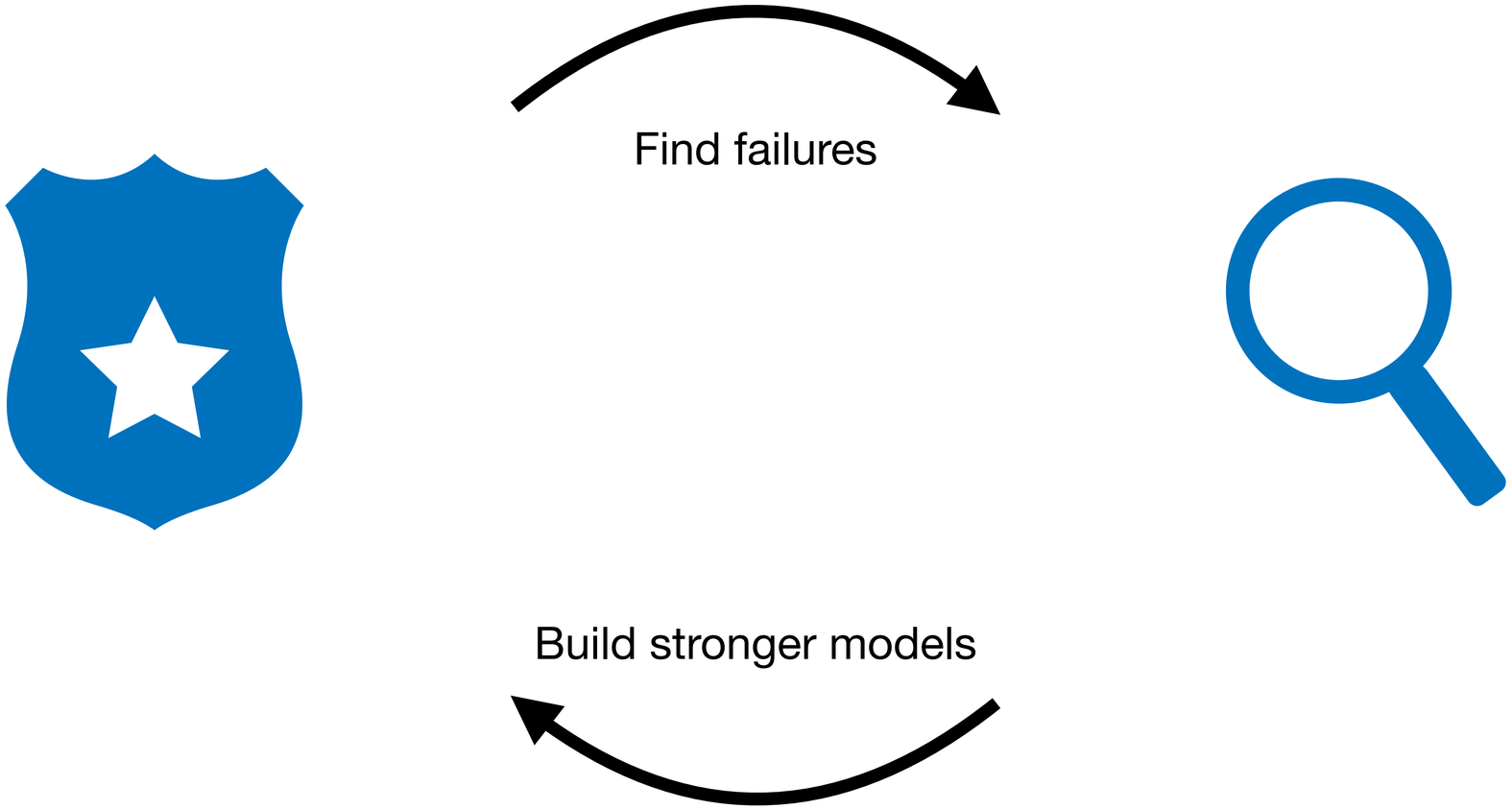
Find failure modes and quantify the probability of failure

$$\mathbb{P}_0(f(X) < \gamma)$$

- Why is this the right problem?
- How do we solve it quickly?

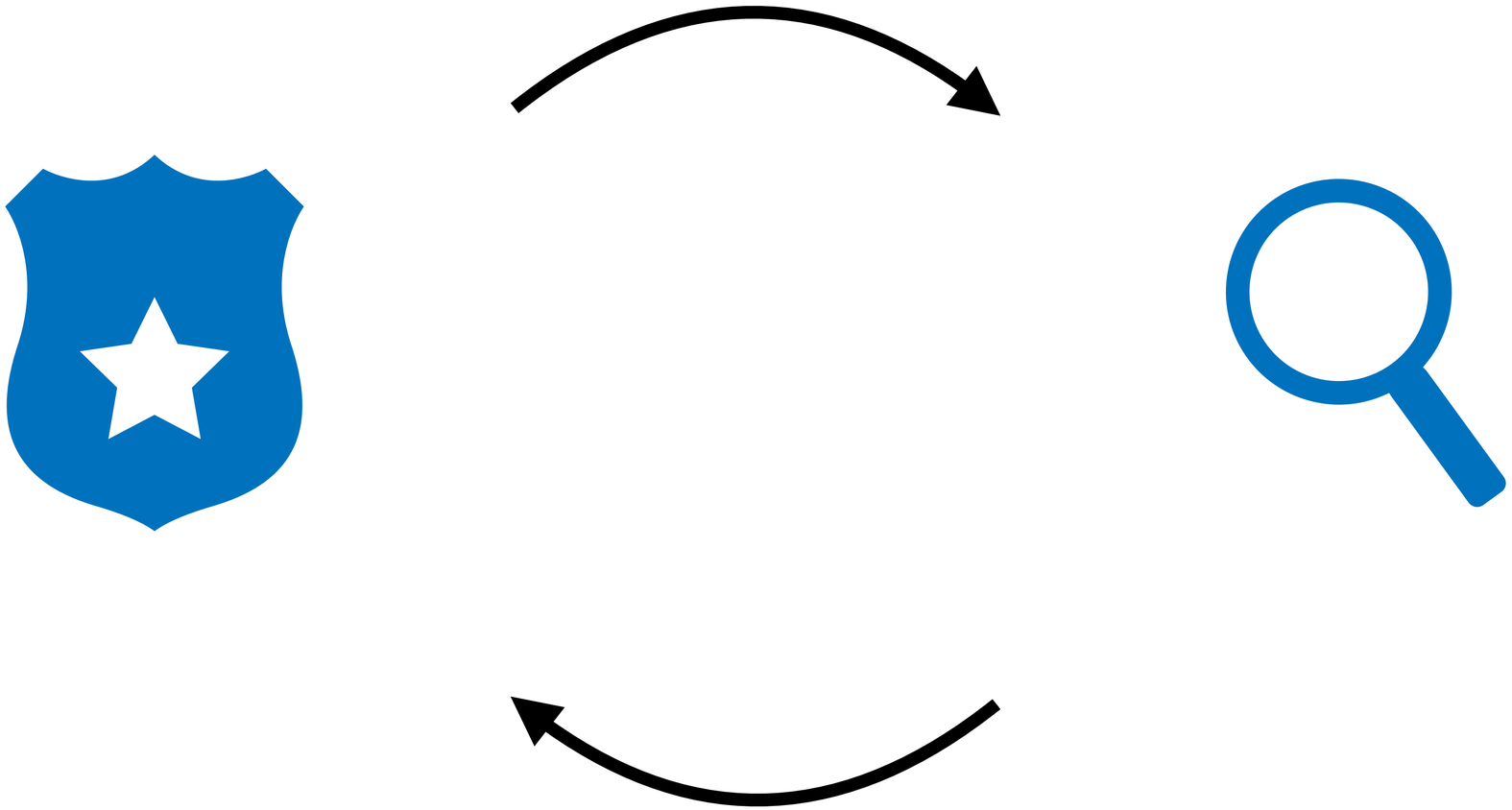
Future directions

Automating the development process



Future directions

Model governance more broadly



Future directions

Model governance more broadly