Dynamic Management of Network Risk from Epidemic Phenomena

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Analyzing Epidemics

- Classic models (SIS, SIR) now generalized to probabilistic models of infection (Ganesh et al. 2005)

- Widely applicable - digital/biological viruses, network router faults, social media influence, etc.

- Control
  - Optimization approaches explicitly include budget constraints (Gourdin et al. 2011, Preciado et al. 2013, Preciado et al. 2014)
  - Our methods also deal with decentralization and robustness

Outline

Model Framework

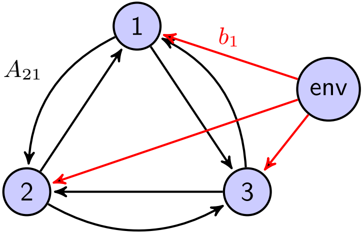
Proposed Approach

Experiments

Conclusions & Future Work

System Dynamics

- SIS epidemic model as a continuous-time Markov process



$$\mathbf{s} = [s_1, s_2, ..., s_N]^T \in \{0,1\}^N$$

$$s_i(t) : \begin{cases} 1 \to 0 \text{ at rate } r_i \\ 0 \to 1 \text{ at rate } \mathbf{e}_i^T A\mathbf{s}(t) + \\ \qquad\qquad\qquad \mathbf{e}_i^T \mathbf{b} s_{env}(t) \end{cases}$$

$$s_{env}(t) : 1 \to 0 \text{ at rate } r_{env}$$

- Instantaneous energy of infection

$$P(\mathbf{1}^T\mathbf{s}(t) > 0) \leq \sqrt{N}\|\mathbf{z}(t)\|_2$$

$$\begin{aligned} \dot{\mathbf{z}}(t) &= D\mathbf{z}(t) + \mathbf{b}e^{-r_{env}t}s_{env}(0), \\ \mathbf{z}(0) &= \mathbf{s}(0), \quad D := A - \operatorname{diag}(\mathbf{r}) \end{aligned}$$

Problem Setup

- Control environmental impact on system via limited budget at discrete intervals
  - Discretize dynamics: $\mathbf{x}(k) := \mathbf{z}(kh)$
  - Control $\mathbf{b}$ w.r.t budget constraints

$$\mathbf{u}(k) = (\mathbf{b} - \mathbf{w}(k))e^{-r_{env}kh}s_{env}(0)$$
$$\mathbf{0} \preceq \mathbf{w}(k) \preceq \mathbf{b}, \;\; \|\mathbf{w}(k)\|_1 \leq c,$$

- Minimize cumulative energy of infection via MPC

$$\int_0^\infty P(\mathbf{1}^T\mathbf{s}(t) > 0)dt \;\leq\; \sqrt{N}\int_0^\infty \|\mathbf{z}(t)\|_2 dt \;\approx\; \sqrt{N}\sum_{k=0}^{T}\|\mathbf{x}(k)\|_2$$

minimize $J_m := \sqrt{N}\sum_{k=m+1}^{T+m}\|\mathbf{x}(k)\|_2$

subject to (dynamics, constraints)

Problem Setup (contd.)

- Centralized solution is inefficient for large N and network connectivity might not be known perfectly
- Decentralization - split system into M (possibly unequal) subsystems
- Robustness - off-diagonal blocks of A are known only within some uncertainty region

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Reduced-Order Models

- Each subsystem models the other subsystems' dynamics through reduced-order models (decentralization/accuracy tradeoff)

- Standard model reduction procedure (e.g. via balanced truncation, Safonov et al. 1988)
  - Procedure outputs compression and expansion operators
  - Analogous to similarity transformation

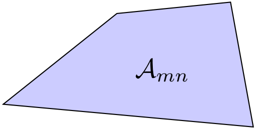
- Local problem for subsystem i (with state $\mathbf{x}_r^i$, control $\mathbf{u}_r^i$):

$$\begin{aligned} &\text{minimize } \; J_m^i := \sqrt{N} \sum_{k=m+1}^{T+m} \|\mathbf{x}_r^i(k)\|_2 \\ &\text{subject to } \; \text{(reduced dynamics, reduced constraints)} \end{aligned}$$

Robust Formulation

- Polytopic/"scenario" uncertainty sets (efficiency/robustness tradeoff)

$$\mathcal{A}_{mn} = \{C | C = \sum_{k=1}^{L_{mn}} \mu_k A_{mn}(k), \;\; \mu_k \geq 0, \;\; \sum_{k=1}^{L_{mn}} \mu_k = 1\}$$



- Straightforward generalization for model reduction via generalized balanced truncation (replace Lyapunov eq. with LMI)

- Robust counterpart for local problem ($\min \sup_{\mathcal{A}} J_m^i$) is an SOCP
  - Requires linearizing dynamics s.t. $\mathbf{x}_r^i(k)$ is affine in A

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- $N = 24$, $M = 3$, equal subsystem sizes, random adjacency matrices and recovery rates

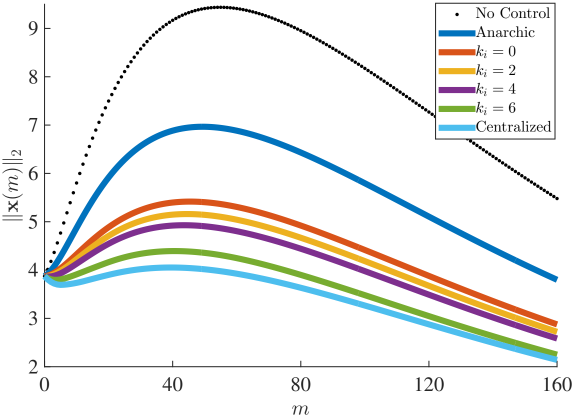
- Environment heals, but at a slower rate than the system
  - $s_{env}(0) = 1$, $r_{env} = 0.2 < -\lambda_i(D) \in [0.33, 1]$

- We vary the order of reduced models, $k_i = \{0, 2, 4, 6, 8\}$

- Compare with no control, anarchy (each subsystem has budget c/M)

Experiments (contd.)

- Cooperation/dynamic budget allocation assuages overshoot
- Larger k_i yields better performance



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- Developed framework for dynamic network protection incorporating budget constraints, decentralization, and robustness to uncertainty

- Tradeoffs between efficiency/robustness and decentralization/optimality

- Many avenues worth further research
  - Uncertainty sets with greater scalability
  - Optimal decentralized schemes for partitioning budgets between subsystems
  - Dynamic network topologies